Asea Brown Boveri Pocket Book

Cornelsen
GIRARDET

Switchgear Manual

10th revised edition

Edited by

Hennig Gremmel for

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Foreword to the tenth edition

More than 50 years after publication of the first edition of the BBC switchgear manual by A. Hoppner, the 10th revised edition is now available as the ABB Calor Emag switchgear manual. As always, it is intended for both experienced switchgear professionals as well as beginners and students.

The 10th edition has been prepared under the direction of the two German ABB companies listed as editors. The products shown as examples to explain the technical statements conform to the practice in the area of switchgear in Germany, and they are products that are manufactured by ABB for the market in this country.

In their efforts to be as up to date as possible, a team of authors comprising experienced engineers from all the relevant areas has described the current and future solutions and technologies. Not only is the technology of switchgear installations and apparatus in the areas of low, medium and high voltage described but related areas such as digital control systems, CAD/CAE methods, project planning, network calculation, electromagnetic compatibility (EMC), etc. are also considered.

In the last few years there has been significant progress in standardization in the implementation of international unified standards. DKE, as the organization responsible for standardization in the area of electrical technology in Germany, has taken account of this development with a new system of numbering DIN and VDE standards. Under this system, since 1993 standards that include safety specifications have their original publication number (e.g. IEC ..., EN ...) as the DIN designation and also a VDE classification number. Section 17 of this book describes this. There, the list of standards shows the complete designations in their current version, however, at the moment not all standards have a DIN designation under the above system. The other sections of the book sometimes also use the complete designation, which however is somewhat cumbersome in daily usage, and sometimes the DIN numbering only and sometimes also the VDE classification, which best indicates the connections.

We would like to thank all involved in the preparation of this book, including the authors of earlier editions, for their valuable suggestions and contributions.

Mannheim and Ratingen, November 1999 / June 2001

ABB Calor Emag
Schaltanlagen AG

ABB Calor Emag
Mittelspannung GmbH

The following have assisted with the preparation of the 10th edition of this handbook with their own contributions or by editing sections:
Klaus Arp, Klaus-Dieter Bäsel, Volker Biewendt, Kurt Blüm, Dietmar Borries, Thomas Braun, Andreas Breuer, Reiner Christmann, Joachim Dobberstein, Karl-Heinz Fettback-Heyden, Franz Fretz, Walter Gentz, Hennig Gremmel, Michael Häussler, Kurt Haneke, Karl-Heinz Hartung, Erich Hebach, Ralf Heinemeyer, Erwin Himmer, Bernhard Hof, Peter Hoyer, Horst Issing, Eckhard Junghahn, Gerald Kopatsch, Ralf Krumm, Martin Lauersdorf, Manfred Mathis, Manfred F. Nehring, Gerhard Noller, Wilhelm Oehmig, Peter Peknice, Horst Plettner, Rolf Pöhlmann, Hans-Gerd Post, Helge Postler, Volker Rees, Peter Reinhardt, Ulrich Reininghaus, Heinrich Remde, Peter Röhrig, Eberhard Rörich, Rolf Rotterdam, Harald Schippel, Gerhard Schmidt, Joachim Schneider, Reinhard Scholz, Herrmann Seidl, Hans Joachim Straube, Woftgang Tettenbom, Andreas Underbrink, Hans Unger, Gerhard Voss, Rolf Wittkämper, Thomas Wotschokowsky, Benjamin Weber, Bodo Zahn, Claus-Dieter Ziebell, Werner Zimmerli, Werner Zimmermann.

Brief Overview

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8 Switchgear and Switchgear Installations for High-Voltage up to and including 52 kV (Medium Voltage)

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1 Fundamental Physical and Technical Terms

1.1 Units of physical quantities

1.1.1 The International System of Units (SI)

The statutory units of measurement are[1)]

1. the basic units of the International System of Units (SI units) for the basic quantities length, mass, time, electric current, thermodynamic temperature and luminous intensity,
2. the units defined for the atomic quantities of quantity of substance, atomic mass and energy,
3. the derived units obtained as products of powers of the basic units and atomic units through multiplication with a defined numerical factor,
4. the decimal multiples and sub-multiples of the units stated under 1-3.

Table 1-1

Basic SI units

Quantity	Units Symbol	Units Name
Length	m	metre
Mass	kg	kilogramme
Time	s	second
Electric current	A	ampere
Thermodynamic temperature	K	kelvin
Luminous intensity	cd	candela
Atomic units		
Quantity of substance	mol	mole

Table 1-2
Decimals
Multiples and sub-multiples of units

Decimal power	Prefix	Symbol			
10^{12}	Tera	T	10^{-2}	Zenti	c
10^{9}	Giga	G	10^{-3}	Milli	m
10^{6}	Mega	M	10^{-6}	Mikro	µ
10^{3}	Kilo	k	10^{-9}	Nano	n
10^{2}	Hekto	h	10^{-12}	Piko	p
10^{1}	Deka	da	10^{-15}	Femto	f
10^{-1}	Dezi	d	10^{-18}	Atto	a

[1)] DIN 1301

Table 1-3

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
1	**Length, area, volume**						
1.1	Length	metre	m				see Note to No. 1.1
1.2	Area	square metre	m^2				
				are	a	$1\ a = 10^2\ m^2$	for land measurement only
				hectare	ha	$1\ ha = 10^4\ m^2$	
1.3	Volume	cubic metre	m^3				
				litre	l	$1\ l = 1\ dm^3 = 10^{-3}\ m^3$	
1.4	Reciprocal length	reciprocal metre	1/m				
				dioptre	dpt	1 dpt = 1/m	only for refractive index of optical systems
1.5	Elongation	metre per metre	m/m				Numerical value of elongation often expressed in per cent

1) See also notes to columns 3 and 4 and to column 7 on page 15.

(continued)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)] Name	Symbol	Other units Name	Symbol	Relationship[1)]	Remarks
2 Angle							
2.1	Plane angle (angle)	radian	rad			1 rad = 1 m/m	see DIN 1315. In calculation the unit rad as a factor can be replaced by numerical 1. (applies to the bracketed rows from 1 rad = 1 m/m through second)
				full angle		1 full angle = 2π rad	
				right angle	∟	1 ∟ = $\frac{\pi}{2}$ rad	
				degree	°	1 ° = $\frac{\pi}{180}$ rad	
				minute	'	1' = 1°/60	
				second	"	1" = 1'/60	
				gon	gon	1 gon = $\frac{\pi}{200}$ rad	
2.2	Solid angle	steradian	sr			1 sr = $1 m^2/m^2$	see DIN 1315

1) See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued)*

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
3	**Mass**						
3.1	Mass	kilogramme	kg				Units of weight used as terms for mass in expressing quantities of goods are the units of mass, see DIN 1305
				gramme	g	1 g = 10^{-3} kg	At the present state of measuring technology the 3-fold standard deviation for the relationship for u given in col. 7 is ± $3 \cdot 10^{-32}$ kg.
				tonne	t	1 t = 10^{3} kg	
				atomic mass unit	u	1 u = $1.66053 \cdot 10^{-27}$ kg	
				metric carat	Kt	1 Kt = $0.2 \cdot 10^{-3}$ kg	only for gems
3.2	Mass per unit length	kilogramme per metre	kg/m				
				Tex	tex	1 tex = 10^{-6} kg/m = 1 g/km	only for textile fibres and yarns, see DIN 60905 Sheet 1

[1)] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued*)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1]		Other units		Relationship[1]	Remarks
		Name	Symbol	Name	Symbol		
3.3	Density	kilogramme per cubic metre	kg/m^3				see DIN 1306
3.4	Specific volume	cubic metre per kilogramme	m^3/kg				see DIN 1306
3.5	Moment of inertia	kilogramme-square metre	$kg\ m^2$				see DIN 5497 and Note to No. 3.5

[1] See also notes to columns 3 and 4 and to column 7 on page 15.

(continued)

Table 1-3 (continued)

List of units

1 No.	*2* Quantity	*3* SI unit[1)] Name	*4* SI unit[1)] Symbol	*5* Other units Name	*6* Other units Symbol	*7* Relationship[1)]	*8* Remarks
4	**Time**						
4.1	Time	second	s	minute hour day year	min h d a	1 min = 60 s 1 h = 60 min 1 d = 24 h	see DIN 1355 In the power industry a year is taken as 8760 hours. See also Note to No. 4.1.
4.2	Frequency	hertz	Hz			1 Hz = 1/s	1 hertz is equal to the frequency of a periodic event having a duration of 1 s.
4.3	Revolutions per second	reciprocal second	1/s	reciprocal minute	1/min	1/min = 1/(60 s)	If it is defined as the reciprocal of the time of revolution, see DIN 1355.

[1)] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued*)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
		SI unit[1]		Other units			
No.	Quantity	Name	Symbol	Name	Symbol	Relationship[1]	Remarks
4.4	Cyclic frequency	reciprocal second	1/s				
4.5	Velocity	metre per second	m/s				
				kilometre per hour	km/h	1 km/h = $\frac{1}{3.6}$ m/s	
4.6	Acceleration	metre per second squared	m/s^2				
4.7	Angular velocity	radian per second	rad/s				
4.8	Angular acceleration	radian per second squared	rad/s^2				

[1] See also notes to columns 3 and 4 and to column 7 on page 15.

(continued)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
5	**Force, energy, power**						
5.1	Force	newton	N			$1\ N = 1\ kg\ m/s^2$	Units of weight as a quantity of force are the units of force, see DIN 1305.
5.2	Momentum	newton-second	Ns			$1\ Ns = 1\ kg\ m/s$	
5.3	Pressure	pascal	Pa			$1\ Pa = 1\ N/m^2$	see Note to columns 3 and 4
				bar	bar	$1\ bar = 10^5\ Pa$	see DIN 1314

1) See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued)*

Table 1-3 (continued)

List of units

1	2	3	4	5	6	7	8
No.	Quantity	SI unit[1]		Other units		Relationship[1]	Remarks
		Name	Symbol	Name	Symbol		
5.4	Mechanical stress	newton per square metre, pascal	N/m^2, Pa			1 Pa = 1 N/m^2	In many technical fields it has been agreed to express mechanical stress and strength in N/mm^2. 1 N/mm^2 = 1 MPa.
5.5	Energy, work, quantity of heat	joule	J			1 J = 1 Nm = 1 Ws = 1 kg m^2/s^2	see DIN 1345
				kilowatt-hour electron volt	kWh eV	1 kWh = 3.6 MJ 1 eV = $1.60219 \cdot 10^{-19}$ J	At the present state of measuring technology the 3-fold standard deviation for the relationship given in col. 7 is $\pm 2 \cdot 10^{-24}$ J.
5.6	Torque	newton-metre	Nm			1 Nm = 1 J = 1 Ws	
5.7	Angular momentum	newton-second-metre	Nsm			1 Nsm = 1 kg m^2/s	

[1] See also notes to columns 3 and 4 and to column 7 on page 15.

(continued)

Table 1-3 (continued)

List of units

1	2	3	4	5	6	7	8
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
5.8	Power energy flow, heat flow	watt	W			1 W = 1 J/s =1 N m/s = 1 VA	The watt is also termed volt-ampere (standard symbol VA) when expressing electrical apparent power, and Var (standard symbol var) when expressing electrical reactive power, see DIN 40110.
6	**Viscometric quantities**						
6.1	Dynamic viscosity	pascal-second	Pas			1 Pas = 1 Ns/m^2 = 1 kg/(sm)	see DIN 1342
6.2	Kinematic viscosity	square metre per second	m^2/s				see DIN 1342

[1)] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued)*

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1]		Other units		Relationship[1]	Remarks
		Name	Symbol	Name	Symbol		
7	**Temperature and heat**						
7.1	Temperature	kelvin	K				Thermodynamic temperature; see Note to No. 7.1 and DIN 1345. Kelvin is also the unit for temperature differences and intervals.
				degree Celsius (centigrade)	° C	The degree Celsius is the special name for kelvin when expressing Celsius temperatures.	Expression of Celsius temperatures and Celsius temperature differences, see Note to No 7.1.
7.2	Thermal diffusivity	square metre per second	m^2/s				see DIN 1341
7.3	Entropy, thermal capacity	joule per kelvin	J/K				see DIN 1345
7.4	Thermal conductivity	watt per kelvin-metre	W/(K m)				see DIN 1341

[1] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued*)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1]		Other units		Relationship[1]	Remarks
		Name	Symbol	Name	Symbol		
7.5	Heat transfer coefficient	watt per kelvin-square metre	$W/(Km^2)$				see DIN 1341
8	**Electrical and magnetic quantities**						
8.1	Electric current, magnetic potential difference	ampere	A				see DIN 1324 and DIN 1325
8.2	Electric voltage, electric potential difference	volt	V			1 V =1 W/A	see DIN 1323
8 3	Electric conductance	siemens	S			1 S = A/V	see Note to columns 3 and 4 and also DIN 1324
8.4	Electric resistance	ohm	Ω			1 Ω = 1/S	see DIN 1324

[1] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued*)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
8.5	Quantity of electricity, electric charge	coulomb	C	 ampere-hour	 Ah	1 C = 1 As 1 Ah = 3600 As	see DIN 1324
8.6	Electric capacitance	farad	F			1 F = 1 C/V	see DIN 1357
8.7	Electric flux density	coulomb per square metre	C/m^2				see DIN 1324
8.8	Electric field strength	volt per metre	V/m				see DIN 1324
8.9	Magnetic flux	weber, volt-second	Wb, Vs			1 Wb = 1 Vs	see DIN 1325
8.10	Magnetic flux density, (induction)	tesla	T			1 T = 1 Wb/m^2	see DIN 1325
8.11	Inductance (permeance)	henry	H			1 H = 1 Wb/A	see DIN 1325

[1)] See also notes to columns 3 and 4 and to column 7 on page 15.

(*continued*)

Table 1-3 (continued)

List of units

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
No.	Quantity	SI unit[1)]		Other units		Relationship[1)]	Remarks
		Name	Symbol	Name	Symbol		
8.12	Magnetic field intensity	ampere per metre	A/m				see DIN 1325
9	**Photometric quantities**						
9.1	Luminous intensity	candela	cd				see DIN 5031 Part 3. The word candela is stressed on the 2nd syllable.
9.2	Luminance	candela per square metre	cd/m²				see DIN 5031 Part 3
9.3	Luminous flux	lumen	lm			1 lm = 1 cd · sr	see DIN 5031 Part 3
9.4	Illumination	lux	lx			1 lx = 1 lm/m²	see DIN 5031 Part 3

[1)] See also notes to columns 3 and 4 and to column 7 on page 15.

Notes to Table 1-3

To column 7:

A number having the last digit in bold type denotes that this number is defined by agreement (see DIN 1333).

To No. 1.1:

The nautical mile is still used for marine navigation (1 nm = 1852 m). For conversion from inches to millimetres see DIN 4890, DIN 4892, DIN 4893.

To No. 3.5:

When converting the so-called “flywheel inertia GD^2” into a mass moment of inertia J, note that the numerical value of GD^2 in kp m^2 is equal to four times the numerical value of the mass moment of inertia J in kg m^2.

To No. 4.1:

Since the year is defined in different ways, the particular year in question should be specified where appropriate.

3 h always denotes a time span (3 hours), but 3^h a moment in time (3 o’clock). When moments in time are stated in mixed form, e.g. $2^h25^m3^s$, the abbreviation min may be shortened to m (see DIN 1355).

To No. 7.1:

The (thermodynamic) temperature *(T)*, also known as “absolute temperature”, is the physical quantity on which the laws of thermodynamics are based. For this reason, only this temperature should be used in physical equations. The unit kelvin can also be used to express temperature differences.

Celsius (centigrade) temperature *(t)* is the special difference between a given thermodynamic temperature T and a temperature of $T_0 = 273.15$ K.

Thus,

$$t = T - T_0 = T - 273.15 \text{ K}. \quad (1)$$

When expressing Celsius temperatures, the standard symbol °C is to be used.

The difference Δ t between two Celsius temperatures, e. g. the temperatures $t_1 = T_1 - T_0$ and $t_2 = T_2 - T_0$, is

$$\Delta t = t_1 - t_2 = T_1 - T_2 = \Delta T \quad (2)$$

A temperature difference of this nature is no longer referred to the thermodynamic temperature T_0, and hence is not a Celsius temperature according to the definition of Eq. (1).

However, the difference between two Celsius temperatures may be expressed either in kelvin or in degrees Celsius, in particular when stating a range of temperatures, e. g. (20 ± 2) °C

Thermodynamic temperatures are often expressed as the sum of T_0 and a Celsius temperature t, i. e. following Eq. (1)

$$T = T_0 + t \quad (3)$$

and so the relevant Celsius temperatures can be put in the equation straight away. In this case the kelvin unit should also be used for the Celsius temperature (i. e. for the “special thermodynamic temperature difference”). For a Celsius temperature of 20 °C, therefore, one should write the sum temperature as

$$T = T_0 + t = 273.15 \text{ K} + 20 \text{ K} = 293.15 \text{ K} \quad (4)$$

1.1.2 Other units still in common use; metric, British and US measures

Some of the units listed below may be used for a limited transition period and in certain exceptional cases. The statutory requirements vary from country to country.

ångström	Å	length	1 Å = 0.1 nm = 10^{-10}m
atmosphere physical	atm	pressure	1 atm = 101 325 Pa
atmosphere technical	at, ata	pressure	1 at = 98 066.5 Pa
British thermal unit	Btu	quantity of heat	1 Btu ≈ 1055.056 J
calorie	cal	quantity of heat	1 cal = 4.1868 J
centigon	c	plane angle	1 c = 1 cgon = $5\,\pi \cdot 10^{-5}$ rad
degree	deg, grd	temperature difference	1 deg = 1 K
degree fahrenheit	°F	temperature	$T_K = 273.15 + (5/9) \cdot (t_F - 32)$
dyn	dyn	force	1 dyn = 10^{-5} N
erg	erg	energy	1 erg = 10^{-7} J
foot	ft	length	1 ft = 0.3048 m
gallon (UK)	gal (UK)	volume	1 gal (UK) ≈ $4.54609 \cdot 10^{-3}$ m³
gallon (US)	gal (US)	liquid volume	1 gal (US) ≈ $3.78541 \cdot 10^{-3}$ m³
gauss	G.Gs	magnetic flux density	1 G = 10^{-4}T
gilbert	Gb	magnetic potential difference	1 Gb = $(10/4\,\pi)$ A
gon	g	plane angle	1 g = 1 gon = $5\,\pi \cdot 10^{-3}$ rad
horsepower	hp	power	1 hp ≈ 745.700 W
hundredweight (long)	cwt	mass	1 cwt ≈ 50.8023 kg
inch (inches)	in, ″	length	1 in = 25.4 mm = $254 \cdot 10^{-4}$ m
international ampere	A_{int}	electric current	1 A_{int} ≈ 0.99985 A
international farad	F_{int}	electrical capacitance	1 F_{int} = (1/1.00049) F
international henry	H_{int}	inductance	1 H_{int} = 1.00049 H
international ohm	Ω_{int}	electrical resistance	1 Ω_{int} = 1.00049 Ω
international volt	V_{int}	electrical potential	1 V_{int} = 1.00034 V
international watt	W_{int}	power	1 W_{int} ≈ 1.00019 W
kilogramme-force, kilopond	kp, kgf	force	1 kp = 9.80665 N ≈ 10 N

Unit of mass	ME	mass	1 ME = 9.80665 kg
maxwell	M, Mx	magnetic flux	1 M = 10 nWb = 10^{-8} Wb
metre water column	mWS	pressure	1 mWS = 9806.65 PA ≈ 0,1 bar
micron	µ	length	1 µ = 1 µm = 10^{-6} m
millimetres of mercury	mm Hg	pressure	1 mm Hg ≈ 133.322 Pa
milligon	cc	plane angle	1 cc = 0.1 mgon = $5 \pi \cdot 10^{-7}$ rad
oersted	Oe	magnetic field strength	10e = (250/π) A/m
Pferdestärke, cheval-vapeur	PS, CV	power	1 PS = 735.49875 W
Pfund	Pfd	mass	1 Pfd = 0.5 kg
pieze	pz	pressure	1 pz = 1 mPa = 10^{-3} Pa
poise	P	dynamic viscosity	1 P = 0.1 Pa · s
pond, gram			
-force	p, gf	force	1 p = $9.80665 \cdot 10^{-3}$ N ≈ 10 mN
pound[1]	lb	mass	1 lb ≈ 0.453592 kg
poundal	pdl	force	1 pdl ≈ 0.138255 N
poundforce	lbf	force	1 lbf ≈ 4.44822 N
sea mile, international	n mile	length (marine)	1 n mile = 1852 m
short hundredweight	sh cwt	mass	1 sh cwt ≈ 45.3592 kg
stokes	St	kinematic viscosity	1 St = 1 cm^2/s = 10^{-4} m^2/s
torr	Torr	pressure	1 Torr ≈ 133.322 Pa
typographical point	p	length (printing)	1 p = (1.00333/2660) m ≈ 0.4 mm
yard	yd	length	1 yd = 0.9144 m
Zentner	z	mass	1 z = 50 kg

[1] UK and US pounds avoirdupois differ only after the sixth decimal place.

Table 1-4

Metric, British and US linear measure

Metric units of length					British and US units of length				
Kilometre	Metre	Decimetre	Centimetre	Millimetre	Mile	Yard	Foot	Inch	Mil
km	m	dm	cm	mm	mile	yd	ft	in or "	mil
1	1 000	10 000	100 000	1 000 000	0.6213	1 093.7	3 281	39 370	$3\,937 \cdot 10^{4}$
0.001	1	10	100	1 000	$0.6213 \cdot 10^{-3}$	1.0937	3.281	39.370	39 370
0.0001	0.1	1	10	100	$0.6213 \cdot 10^{-4}$	0.1094	0.3281	3.937	3 937.0
0.00001	0.01	0.1	1	10	$0.6213 \cdot 10^{-5}$	0.01094	0.03281	0.3937	393.70
0.000001	0.001	0.01	0.1	1	$0.6213 \cdot 10^{-6}$	0.001094	0.003281	0.03937	39.37
1.60953	1 609.53	16 095.3	160 953	1 609 528	1	1 760	5 280	63 360	$6\,336 \cdot 10^{4}$
0.000914	0.9143	9.1432	91.432	914.32	$0.5682 \cdot 10^{-3}$	1	3	36	36 000
$0.305 \cdot 10^{-3}$	0.30479	3.0479	30.479	304.79	$0.1894 \cdot 10^{-3}$	0.3333	1	12	12 000
$0.254 \cdot 10^{-4}$	0.02539	0.25399	2.53997	25.3997	$0.158 \cdot 10^{-4}$	0.02777	0.0833	1	1 000
$0.254 \cdot 10^{-7}$	$0.254 \cdot 10^{-4}$	$0.254 \cdot 10^{-3}$	0.00254	0.02539	$0.158 \cdot 10^{-7}$	$0.0277 \cdot 10^{-3}$	$0.0833 \cdot 10^{-3}$	0.001	1

Special measures: 1 metric nautical mile = 1852 m; 1 Brit. or US nautical mile = 1855 m; 1 metric land mile = 7500 m; 1 micron (μ) = 1/1000 mm = 10 000 Å

Table 1-5

Metric, British and US square measure

Metric units of area					British and US units of area				
Square kilometres	Square metre	Square decim.	Square centim.	Square millim.	Square mile	Square yard	Square foot	Square inch	Circular mils
km^2	m^2	dm^2	cm^2	mm^2	sq.mile	sq.yd	sq.ft	sq.in	cir.mils
1	$1 \cdot 10^{6}$	$100 \cdot 10^{6}$	$100 \cdot 10^{8}$	$100 \cdot 10^{10}$	0.386013	$1\,196 \cdot 10^{3}$	$1076 \cdot 10^{4}$	$1\,550 \cdot 10^{6}$	$197.3 \cdot 10^{13}$
$1 \cdot 10^{-6}$	1	100	10 000	1 000 000	$0.386 \cdot 10^{-6}$	1.1959	10.764	1 550	$197.3 \cdot 10^{7}$
$1 \cdot 10^{-8}$	$1 \cdot 10^{-2}$	1	100	10 000	$0.386 \cdot 10^{-8}$	0.01196	0.10764	15.50	$197.3 \cdot 10^{5}$
$1 \cdot 10^{-10}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-2}$	1	100	$0.386 \cdot 10^{-10}$	$0.1196 \cdot 10^{-3}$	$0.1076 \cdot 10^{-2}$	0.1550	$197.3 \cdot 10^{3}$
$1 \cdot 10^{-12}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-2}$	1	$0.386 \cdot 10^{-12}$	$0.1196 \cdot 10^{-5}$	$0.1076 \cdot 10^{-4}$	0.00155	1 973
2.58999	2 589 999	$259 \cdot 10^{6}$	$259 \cdot 10^{8}$	$259 \cdot 10^{10}$	1	$30\,976 \cdot 10^{2}$	$27\,878 \cdot 10^{3}$	$40\,145 \cdot 10^{5}$	$5\,098 \cdot 10^{12}$
$0.8361 \cdot 10^{-6}$	0.836130	83.6130	8 361.307	836 130.7	$0.3228 \cdot 10^{-6}$	1	9	1296	$1\,646 \cdot 10^{6}$
$9.290 \cdot 10^{-8}$	$9.290 \cdot 10^{-2}$	9.29034	929.034	92 903.4	$0.0358 \cdot 10^{-6}$	0.11111	1	144	$183 \cdot 10^{6}$
$6.452 \cdot 10^{-10}$	$6.452 \cdot 10^{-4}$	$6.452 \cdot 10^{-2}$	6.45162	645.162	$0.2396 \cdot 10^{-9}$	$0.7716 \cdot 10^{-3}$	0.006940	1	$1.27 \cdot 10^{6}$
$506.7 \cdot 10^{-18}$	$506.7 \cdot 10^{-12}$	$506.7 \cdot 10^{-10}$	$506.7 \cdot 10^{-8}$	$506.7 \cdot 10^{-6}$	$0.196 \cdot 10^{-15}$	$0.607 \cdot 10^{-9}$	$0.00547 \cdot 10^{-6}$	$0.785 \cdot 10^{-6}$	1

Special measures:

1 hectare (ha) = 100 are (a)
1 are (a) = 100 m^2
1 Bad. morgen = 56 a = 1.38 acre
1 Prussian morgen = 25.53 a = 0.63 acre
1 Württemberg morgen = 31.52 a = 0.78 acre
1 Hesse morgen = 25.0 a = 0.62 acre
1 Tagwerk (Bavaria) = 34.07 a = 0.84 acre
1 sheet of paper = 86 x 61 cm
gives 8 pieces size A4 or 16 pieces A5
or 32 pieces A6

USA:
1 section (sq.mile) = 64 acres = 2,589 km^2
1 acre = 4840 sq.yds = 40.468 a
1 sq. pole = 30.25 sq.yds = 25.29 m^2

Brit.:
1 acre = 160 sq.poles = 4840 sq.yds = 40.468 a
1 yard of land = 30 acres = 1214.05 a
1 mile of land = 640 acres = 2.589 km^2

Table 1-6

Metric, British and US cubic measures

Metric units of volume				British and US units of volume			US liquid measure		
Cubic metre	Cubic decimetre	Cubic centimetre	Cubic millimetre	Cubic yard	Cubic foot	Cubic inch	Gallon	Quart	Pint
m^3	dm^3	cm^3	mm^3	cu.yd	cu.ft	cu.in	gal	quart	pint
1	1 000	$1\,000 \cdot 10^{3}$	$1\,000 \cdot 10^{6}$	1.3079	35.32	$61 \cdot 10^{3}$	264.2	1 056.8	2 113.6
$1 \cdot 10^{-3}$	1	1 000	$1\,000 \cdot 10^{3}$	$1.3079 \cdot 10^{-3}$	0.03532	61.023	0.2642	1.0568	2.1136
$1 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	1	1 000	$1.3079 \cdot 10^{-6}$	$0.3532 \cdot 10^{-4}$	0.061023	$0.2642 \cdot 10^{-3}$	$1.0568 \cdot 10^{-3}$	$2.1136 \cdot 10^{-3}$
$1 \cdot 10^{-9}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	1	$1.3079 \cdot 10^{-9}$	$0.3532 \cdot 10^{-7}$	$0.610 \cdot 10^{-4}$	$0.2642 \cdot 10^{-6}$	$1.0568 \cdot 10^{-6}$	$2.1136 \cdot 10^{-6}$
0.764573	764.573	764 573	$764\,573 \cdot 10^{3}$	1	27	46 656	202	808	1 616
0.0283170	28.31701	28 317.01	28 317 013	0.037037	1	1 728	7.48224	29.92896	59.85792
$0.1638 \cdot 10^{-4}$	0.0163871	16.38716	16387.16	$0.2143 \cdot 10^{-4}$	$0.5787 \cdot 10^{-3}$	1	0.00433	0.01732	0.03464
$3.785 \cdot 10^{-3}$	3.785442	3 785.442	3 785 442	0.0049457	0.1336797	231	1	4	8
$0.9463 \cdot 10^{-3}$	0.9463605	946.3605	946 360.5	0.0012364	0.0334199	57.75	0.250	1	2
$0.4732 \cdot 10^{-3}$	0.4731802	473.1802	473 180.2	0.0006182	0.0167099	28.875	0.125	0.500	1

Table 1-7

Conversion tables

Millimetres to inches, formula: mm x 0.03937 = inch

mm	0	1	2	3	4	5	6	7	8	9
0		0.03937	0.07874	0.11811	0.15748	0.19685	0.23622	0.27559	0.31496	0.35433
10	0.39370	0.43307	0.47244	0.51181	0.55118	0.59055	0.62992	0.66929	0.70866	0.74803
20	0.78740	0.82677	0.86614	0.90551	0.94488	0.98425	1.02362	1.06299	1.10236	1.14173
30	1.18110	1.22047	1.25984	1.29921	1.33858	1.37795	1.41732	1.45669	1.49606	1.53543
40	1.57480	1.61417	1.65354	1.69291	1.73228	1.77165	1.81102	1.85039	1.88976	1.92913
50	1.96850	2.00787	2.04724	2.08661	2.12598	2.16535	2.20472	2.24409	2.28346	2.32283

Inches to millimetres, formula: inches x 25.4 = mm

inch	0	1	2	3	4	5	6	7	8	9
0		25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
10	254.0	279.4	304.8	330.2	355.6	381.0	406.4	431.8	457.2	482.6
20	508.0	533.4	558.8	584.2	609.6	635.0	660.4	685.8	711.2	736.6
30	762.0	787.4	812.8	838.2	863.6	889.0	914.4	939.8	965.2	990.8
40	1 016.0	1 041.4	1 066.8	1 092.2	1 117.6	1 143.0	1 168.4	1 193.8	1 219.2	1 244.6
50	1 270.0	1 295.4	1 320.8	1 246.2	1 371.6	1 397.0	1 422.4	1 447.8	1 473.2	1 498.6

Fractions of inch to millimetres

inch	mm	inch	mm	inch	mm	inch	mm	inch	mm
1/64	0.397	7/32	5.556	27/64	10.716	5/8	15.875	53/64	21.034
1/32	0.794	15/64	5.953	7/16	11.112	41/64	16.272	27/32	21.431
3/64	1.191	1/4	6.350	29/64	11.509	21/32	16.669	55/64	21.828
1/16	1.587	17/64	6.747	15/32	11.906	43/64	17.066	7/8	22.225
5/64	1.984	9/32	7.144	31/64	12.303	11/16	17.462	57/64	22.622
3/32	2.381	19/64	7.541	1/2	12.700	45/64	17.859	29/32	23.019
7/64	2.778	5/6	7.937	33/64	13.097	23/32	18.256	59/64	23.416
1/8	3.175	21/64	8.334	17/32	13.494	47/64	18.653	15/16	23.812
9/64	3.572	11/32	8.731	35/64	13.891	3/4	19.050	61/64	24.209
5/32	3.969	23/64	9.128	9/16	14.287	49/64	19.447	31/32	24.606
11/64	4.366	3/8	9.525	37/64	14.684	25/32	19.844	63/64	25.003
3/16	4.762	25/64	9.922	19/32	15.081	51/64	20.241	1	25.400
13/64	5.159	13/32	10.319	39/64	15.478	13/16	20.637	2	50.800

1.1.3 Fundamental physical constants

General gas constant: $R = 8.3166\ J\ K^{-1}\ mol^{-1}$
is the work done by one mole of an ideal gas under constant pressure (1013 hPa) when its temperature rises from 0 °C to 1 °C.

Avogadro's constant: N_A (Loschmidt's number N_L): $N_A = 6.0225 \cdot 10^{23}\ mol^{-1}$
number of molecules of an ideal gas in one mole.
When $V_m = 2.2414 \cdot 10^4\ cm^3 \cdot mol^{-1}$: $N_A/V_m = 2.686\ 10^{19}\ cm^{-3}$.

Atomic weight of the carbon atom: $^{12}C = 12.0000$
is the reference quantity for the relative atomic weights of fundamental substances.

Base of natural logarithms: $e = 2.718282$

Bohr's radius: $r_1 = 0.529 \cdot 10^{-8}$ cm
radius of the innermost electron orbit in Bohr's atomic model

Boltzmann's constant: $k = \frac{R}{N_A} = 1.38 \cdot 10^{-23}\ J \cdot K^{-1}$

is the mean energy gain of a molecule or atom when heated by 1 K.

Elementary charge: $e_o = F/N_A = 1.602 \cdot 10^{-19}$ As
is the smallest possible charge a charge carrier (e.g. electron or proton) can have.

Electron-volt: $eV = 1.602 \cdot 10^{-19}$ J

Energy mass equivalent: $8.987 \cdot 10^{13}\ J \cdot g^{-1} = 1.78 \cdot 10^{-27}\ g\ (MeV)^{-1}$
according to Einstein, following $E = m \cdot c^2$, the mathematical basis for all observed transformation processes in sub-atomic ranges.

Faraday's constant: $F = 96\,480\ As \cdot mol^{-1}$
is the quantity of current transported by one mole of univalent ions.

Field constant, electrical: $\varepsilon_o = 0.885419 \cdot 10^{-11}\ F \cdot m^{-1}$.
a proportionality factor relating charge density to electric field strength.

Field constant, magnetic: $\mu_0 = 4 \cdot \pi \cdot 10^{-7}\ H \cdot m^{-1}$
a proportionality factor relating magnetic flux density to magnetic field strength.

Gravitational constant: $\gamma = 6.670 \cdot 10^{-11}\ m^4 \cdot N^{-1} \cdot s^{-4}$
is the attractive force in N acting between two masses each of 1 kg weight separated by a distance of 1 m.

Velocity of light in vacuo: $c = 2.99792 \cdot 10^8\ m \cdot s^{-1}$
maximum possible velocity. Speed of propagation of electro-magnetic waves.

Mole volume: $V_m = 22\,414\ cm^3 \cdot mol^{-1}$
the volume occupied by one mole of an ideal gas at 0 °C and 1013 mbar. A mole is that quantity (mass) of a substance which is numerically equal in grammes to the molecular weight (1 mol H_2 = 2 g H_2)

Planck's constant: $h = 6.625 \cdot 10^{-34}\ J \cdot s$
a proportionality factor relating energy and frequency of a light quantum (photon).

Stefan Boltzmann's radiation constant: $\delta = 5.6697 \cdot 10^{-8}\ W \cdot m^{-2}\ K^{-4}$ relates radiant energy to the temperature of a radiant body. Radiation coefficient of a black body.

Temperature of absolute zero: $T_0 = -273.16$ °C = 0 K.

Wave impedance of space: $\Gamma_0 = 376.73\ \Omega$
coefficient for the H/E distribution with electromagnetic wave propagation.

$$\Gamma_0 = \sqrt{\mu_0/\varepsilon_0} = \mu_0 \cdot c = 1/(\varepsilon_0 \cdot c)$$

Weston standard cadmium cell: $E_0 = 1.0186$ V at 20 °C.

Wien's displacement constant: $A = 0.28978$ cm · K
enables the temperature of a light source to be calculated from its spectrum.

1.2 Physical, chemical and technical values

1.2.1 Electrochemical series

If different metals are joined together in a manner permitting conduction, and both are wetted by a liquid such as water, acids, etc., an electrolytic cell is formed which gives rise to corrosion. The amount of corrosion increases with the differences in potential. If such conducting joints cannot be avoided, the two metals must be insulated from each other by protective coatings or by constructional means. In outdoor installations, therefore, aluminium/copper connectors or washers of copper-plated aluminium sheet are used to join aluminium and copper, while in dry indoor installations aluminium and copper may be joined without the need for special protective measures.

Table 1-8

Electrochemical series, normal potentials against hydrogen, in volts.

1. Lithium	approx. – 3.02	10. Zinc	approx. – 0.77	19. Hydrogen	approx. 0.0
2. Potassium	approx. – 2.95	11. Chromium	approx. – 0.56	20. Antimony	approx. + 0.2
3. Barium	approx. – 2.8	12. Iron	approx. – 0.43	21. Bismuth	approx. + 0.2
4. Sodium	approx. – 2.72	13. Cadmium	approx. – 0.42	22. Arsenic	approx. + 0.3
5. Strontium	approx. – 2.7	14. Thallium	approx. – 0.34	23. Copper	approx. + 0.35
6. Calcium	approx. – 2.5	15. Cobalt	approx. – 0.26	24. Silver	approx. + 0.80
7. Magnesium	approx. – 1.8	16. Nickel	approx. – 0.20	25. Mercury	approx. + 0.86
8. Aluminium	approx. – 1.45	17. Tin	approx. – 0.146	26. Platinum	approx. + 0.87
9. Manganese	approx. – 1.1	18. Lead	approx. – 0.132	27. Gold	approx. + 1.5

If two metals included in this table come into contact, the metal mentioned first will corrode.

The less noble metal becomes the anode and the more noble acts as the cathode. As a result, the less noble metal corrodes and the more noble metal is protected.

Metallic oxides are always less strongly electronegative, i. e. nobler in the electrolytic sense, than the pure metals. Electrolytic potential differences can therefore also occur between metal surfaces which to the engineer appear very little different. Even though the potential differences for cast iron and steel, for example, with clean and rusty surfaces are small, as shown in Table 1-9, under suitable circumstances these small differences can nevertheless give rise to significant direct currents, and hence corrosive attack.

Table 1-9

Standard potentials of different types of iron against hydrogen, in volts

SM steel, clean surface	approx. – 0.40	cast iron, rusty	approx. – 0.30
cast iron, clean surface	approx. – 0.38	SM steel, rusty	approx. – 0.25

1.2.2 Faraday's law

1. The amount m (mass) of the substances deposited or converted at an electrode is proportional to the quantity of electricity $Q = I \cdot t$.
   $m \sim I \cdot t$

2. The amounts m (masses) of the substances converted from different electrolytes by equal quantities of electricity $Q = I \cdot t$ behave as their electrochemical equivalent masses M^*. The equivalent mass M^* is the molar mass M divided by the electrochemical valency n (a number). The quantities M and M^* can be stated in g/mol.

$$m = \frac{M^*}{F} I \cdot t$$

If during electroysis the current I is not constant, the product $I \cdot$ t must be represented by the integral $\int_{t_1}^{t_2} I \, dt$.

The quantity of electricity per mole necessary to deposit or convert the equivalent mass of 1 g/mol of a substance (both by oxidation at the anode and by reduction at the cathode) is equal in magnitude to Faraday's constant (F = 96480 As/mol).

Table 1-10

Electrochemical equivalents[1)]

	Valency *n*	Equivalent mass[2)] g/mol	Quantity precipitated, theoretical g/Ah	Approximate optimum current efficiency %
Aluminium	3	8.9935	0.33558	85 … 98
Cadmium	2	56.20	2.0970	95 … 95
Caustic potash	1	56.10937	2.0036	95
Caustic soda	1	30.09717	1.49243	95
Chlorine	1	35.453	1.32287	95
Chromium	3	17.332	0.64672	—
Chromium	6	8.666	0.32336	10 … 18
Copper	1	63.54	2.37090	65 … 98
Copper	2	31.77	1.18545	97 … 100
Gold	3	65.6376	2.44884	—
Hydrogen	1	1.00797	0.037610	100
Iron	2	27.9235	1.04190	95 … 100
Iron	3	18.6156	0.69461	—
Lead	2	103.595	3.80543	95 … 100
Magnesium	2	12.156	0.45358	—
Nickel	2	29.355	1.09534	95 … 98
Nickel	3	19.57	0.73022	—
Oxygen	2	7.9997	0.29850	100
Silver	1	107.870	4.02500	98 … 100
Tin	2	59.345	2.21437	70 … 95
Tin	4	29.6725	1.10718	70 … 95
Zinc	2	32.685	1.21959	85 … 93

1) Relative to the carbon-12 isotope = 12.000.

2) Chemical equivalent mass is molar mass/valency in g/mol.

Example:

Copper and iron earthing electrodes connected to each other by way of the neutral conductor form a galvanic cell with a potential difference of about 0.7 V (see Table 1-8). These cells are short-circuited via the neutral conductor. Their internal resistance is determined by the earth resistance of the two earth electrodes. Let us say the sum of all these resistances is 10 Ω. Thus, if the drop in "short-circuit emf" relative to the "open-circuit emf" is estimated to be 50 % approximately, a continuous corrosion current of 35 mA will flow, causing the iron electrode to decompose. In a year this will give an electrolytically active quantity of electricity of

$$35\ \text{mA} \cdot 8760\ \frac{\text{h}}{\text{a}} = 306\ \frac{\text{Ah}}{\text{a}}.$$

Since the equivalent mass of bivalent iron is 27.93 g/mol, the annual loss of weight from the iron electrode will be

$$m = \frac{27.93\ \text{g/mol}}{96480\ \text{As/mol}} \cdot 306\ \text{Ah/a} \cdot \frac{3600\ \text{s}}{\text{h}} = 320\ \text{g/a}.$$

1.2.3 Thermoelectric series

If two wires of two different metals or semiconductors are joined together at their ends and the two junctions are exposed to different temperatures, a thermoelectric current flows in the wire loop (Seebeck effect, thermocouple). Conversely, a temperature difference between the two junctions occurs if an electric current is passed through the wire loop (Peltier effect).

The thermoelectric voltage is the difference between the values, in millivolts, stated in Table 1-11. These relate to a reference wire of platinum and a temperature difference of 100 K.

Table 1-11

Thermoelectric series, values in mV, for platinum as reference and temperature difference of 100 K

Bismut ll axis	−7.7	Rhodium	0.65
Bismut ⊥ axis	−5.2	Silver	0.67 … 0.79
Constantan	−3.37 … −3.4	Copper	0.72 … 0.77
Cobalt	−1.99 … −1.52	Steel (V2A)	0.77
Nickel	−1.94 … −1.2	Zinc	0.6 … 0.79
Mercury	−0.07 … +0.04	Manganin	0.57 … 0.82
Platinum	± 0	Irdium	0.65 … 0.68
Graphite	0.22	Gold	0.56 … 0.8
Carbon	0.25 … 0.30	Cadmium	0.85 … 0.92
Tantalum	0.34 … 0.51	Molybdenum	1.16 … 1.31
Tin	0.4 … 0.44	Iron	1.87 … 1.89
Lead	0.41 … 0.46	Chrome nickel	2.2
Magnesium	0.4 … 0.43	Antimony	4.7 … 4.86
Aluminium	0.37 … 0.41	Silicon	44.8
Tungsten	0.65 … 0.9	Tellurium	50
Common thermocouples			
Copper/constantan (Cu/const)	up to 500 °C	Nickel chromium/nickel (NiCr/Ni)	up to 1 000 °C
Iron/constantan (Fe/const)	up to 700 °C	Platinum rhodium/ platinum	up to 1 600 °C
Nickel chromium/ constantan	up to 800 °C	Platinum rhodium/ platinum rhodium	up to 1 800 °C

1.2.4 pH value

The pH value is a measure of the "acidity" of aqueous solutions. It is defined as the logarithm to base 10 of the reciprocal of the hydrogen ion concentration CH_3O[1).]

$$pH \equiv -\log CH_3O.$$

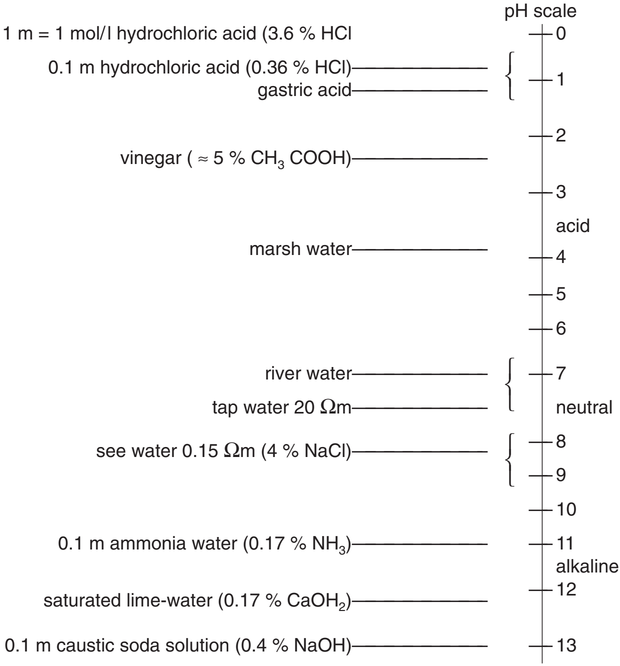


Fig. 1-1

pH value of some solutions

[1)] CH_3O = Hydrogen ion concentration in mol/l.

1.2.5 Heat transfer

Heat content (enthalpy) of a body: $Q = V \cdot \rho \cdot c \cdot \Delta\vartheta$
V volume, ρ density, c specific heat, $\Delta\vartheta$ temperature difference

Heat flow is equal to enthalpy per unit time:

$$\Phi = Q/t$$

Heat flow is therefore measured in watts (1 W = 1 J/s).

Specific heat (specific thermal capacity) of a substance is the quantity of heat required to raise the temperature of 1 kg of this substance by 1 °C. Mean specific heat relates to a temperature range, which must be stated. For values of c and λ, see Section 1.2.7.

Thermal conductivity is the quantity of heat flowing per unit time through a wall 1 m² in area and 1 m thick when the temperatures of the two surfaces differ by 1 °C. With many materials it increases with rising temperature, with magnetic materials (iron, nickel) it first falls to the Curie point, and only then rises (Curie point = temperature at which a ferro-magnetic material becomes non-magnetic, e. g. about 800 °C for Alnico). With solids, thermal conductivity generally does not vary much (invariable only with pure metals); in the case of liquids and gases, on the other hand, it is often strongly influenced by temperature.

Heat can be transferred from a place of higher temperature to a place of lower temperature by

– conduction (heat transmission between touching particles in solid, liquid or gaseous bodies).
– convection (circulation of warm and cool liquid or gas particles).
– radiation (heat transmission by electromagnetic waves, even if there is no matter between the bodies).

The three forms of heat transfer usually occur together.

Heat flow with conduction through a wall:

$$\Phi = \frac{\lambda}{s} \cdot A \cdot \Delta\vartheta$$

A transfer area, λ thermal conductivity, s wall thickness, $\Delta\vartheta$ temperature difference.

Heat flow in the case of transfer by convection between a solid wall and a flowing medium:

$$\Phi = \alpha \cdot A \cdot \Delta\vartheta$$

α heat transfer coefficient, A transfer area, $\Delta\vartheta$ temperature difference.

Heat flow between two flowing media of constant temperature separated by a solid wall:

$$\Phi = k \cdot A \cdot \Delta\vartheta$$

k thermal conductance, A transfer area, $\Delta\vartheta$ temperature difference.

In the case of plane layered walls perpendicular to the heat flow, the thermal conductance coefficient k is obtained from the equation

$$\frac{1}{k} = \frac{1}{\alpha_1} + \sum \frac{s_n}{\lambda_n} + \frac{1}{\alpha_2}$$

Here, α_1 and α_2 are the heat transfer coefficients at either side of a wall consisting of n layers of thicknesses s_n and thermal conductivities λ_n.

For two parallel black surfaces of equal size the heat flow exchanged by radiation is

$$\Phi_{12} = \sigma \cdot A(T_1^4 - T_2^4)$$

With grey radiating surfaces having emissivities of ε_1 and ε_2, it is

$$\Phi_{12} = C_{12} \cdot A\,(T_1^4 - T_2^4)$$

$\sigma = 5.6697 \cdot 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$ radiation coefficient of a black body (Stefan Boltzmann's constant), A radiating area, T absolute temperature.

Index 1 refers to the radiating surface, Index 2 to the radiated surface.

C_{12} is the effective radiation transfer coefficient. It is determined by the geometry and emissivity ε of the surface.

Special cases: $A_1 \ll A_2$ $\qquad C_{12} = \sigma \cdot \varepsilon_1$

$A_1 \approx A_2$ $\qquad C_{12} = \dfrac{\sigma}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$

A_2 includes A_1 $\qquad C_{12} = \dfrac{\sigma}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2} \cdot \left(\dfrac{1}{\varepsilon_2} - 1\right)}$

Table 1-12

Emissivity ε (average values $\vartheta < 200$ °C)

Black body	1	Oil	0.82
Aluminium, bright	0.04	Paper	0.85
Aluminium, oxidized	0.5	Porcelain, glazed	0.92
Copper, bright	0.05	Ice	0.96
Copper, oxidized	0.6	Wood (beech)	0.92
Brass, bright	0.05	Roofing felt	0.93
Brass, dull	0.22	Paints	0.8-0.95
Steel, dull, oxidized	0.8	Red lead oxide	0.9
Steel, polished	0.06	Soot	0.94

Table 1-13

Heat transfer coefficients α in $W/(m^2 \cdot K)$ (average values)		
Natural air movement in a closed space		
Wall surfaces		10
Floors, ceilings:	in upward direction	7
	in downward direction	5
Force-circulated air		
Mean air velocity w = 2 m/s		20
Mean air velocity w > 5 m/s		$6.4 \cdot w^{0.75}$

1.2.6 Acoustics, noise measurement, noise abatement

Perceived sound comprises the mechanical oscillations and waves of an elastic medium in the frequency range of the human ear of between 16 Hz and 20 000 Hz. Oscillations below 16 Hz are termed infrasound and above 20 000 Hz ultrasound. Sound waves can occur not only in air but also in liquids (water-borne sound) and in solid bodies (solid-borne sound). Solid-borne sound is partly converted into audible air-borne sound at the bounding surfaces of the oscillating body. The frequency of oscillation determines the pitch of the sound. The sound generally propagates spherically from the sound source, as longitudinal waves in gases and liquids and as longitudinal and transverse waves in solids.

Sound propagation gives rise to an alternating pressure, the root-mean-square value of which is termed the sound pressure p. It decreases approximately as the square of the distance from the sound source. The sound power P is the sound energy flowing through an area in unit time. Its unit of measurement is the watt.

Since the sensitivity of the human ear is proportional to the logarithm of the sound pressure, a logarithmic scale is used to represent the sound pressure level as loudness.

The *sound pressure level* L is measured with a sound level metre as the logarithm of the ratio of sound pressure to the reference pressure p_o, see DIN 35 632

$$L = 20 \lg \frac{p}{p_o} \text{ in dB.}$$

Here: p_o reference pressure, roughly the audible threshold at 1000 Hz.

$$p_o = 2 \cdot 10^{-5}\ N/m^2 = 2 \cdot 10^{-4}\ \mu bar$$

p = the root-mean-square sound pressure

Example:

$p = 2 \cdot 10^{-3}\ N/m^2$ measured with a sound level metre, then

$$\text{sound level } L = 20 \lg \frac{2 \cdot 10^{-3}}{2 \cdot 10^{-5}} = 40 \text{ dB.}$$

The *loudness* of a sound can be measured as DIN loudness (DIN 5045) or as the weighted sound pressure level. DIN loudness (λ DIN) is expressed in units of DIN phon.

The weighted sound pressure levels L_A, L_B, L_C, which are obtained by switching in defined weighting networks A, B, C in the sound level metre, are stated in the unit dB (decibel). The letters A, B and C must be added to the units in order to distinguish the different values, e. g. dB (A). According to an ISO proposal, the weighted sound pressure L_A in dB (A) is recommended for expressing the loudness of machinery noise. DIN loudness and the weighted sound pressure level, e.g. as recommended in IEC publication 123, are related as follows: for all numerical values above 60 the DIN loudness in DIN phon corresponds to the sound pressure level LB in dB (B), for all numerical values between 30 and 60 to the sound pressure level LA in dB (A). All noise level values are referred to a sound pressure of $2 \cdot 10^{-5}$ N/m^2.

According to VDI guideline 2058, the acceptable loudness of noises must on average not exceed the following values at the point of origin:

Area	Daytime (6–22 hrs) dB (A)	Night-time (22–6 hrs) dB (A)
Industrial	70	70
Commercial	65	50
Composite	60	45
Generally residential	55	40
Purely residential	50	35
Therapy (hospitals, etc.)	45	35

Short-lived, isolated noise peaks can be disregarded.

Disturbing noise is propagated as air- and solid-borne sound. When these sound waves strike a wall, some is thrown back by reflection and some is absorbed by the wall. Air-borne noise striking a wall causes it to vibrate and so the sound is transmitted into the adjacent space. Solid-borne sound is converted into audible air-borne sound by radiation from the bounding surfaces. Ducts, air-shafts, piping systems and the like can transmit sound waves to other rooms. Special attention must therefore be paid to this at the design stage.

There is a logarithmic relationship between the sound pressure of several sound sources and their total loudness.

Total loudness of several sound sources:

A doubling of equally loud sound sources raises the sound level by 3 dB (example: 3 sound sources of 85 dB produce 88 dB together). Several sound sources of different loudness produce together roughly the loudness of the loudest sound source. (Example: 2 sound sources of 80 and 86 dB have a total loudness of 87 dB). In consequence: with 2 equally loud sound sources attenuate both of them, with sound sources of different loudness attentuate only the louder.

An increase in level of 10 dB signifies a doubling, a reduction of 10 dB a halving of the perceived loudness.

In general, noises must be kept as low as possible at their point of origin. This can often be achieved by enclosing the noise sources.

Sound can be reduced by natural means. The most commonly used sound-absorbent materials are porous substances, plastics, cork, glass fibre and mineral wool, etc. The main aim should be to reduce the higher-frequency noise components. This is also generally easier to achieve than eliminating the lower-frequency noise.

When testing walls and ceilings for their behaviour regarding air-borne sound, one determines the difference "D" in sound level "L" for the frequency range from 100 Hz to 3200 Hz.

$$D = L1 - L2 \text{ in dB where } L = 20 \lg \frac{p}{p_o} \text{ dB}$$

L_1 = sound level in room containing sound source

L_2 = sound level in room receiving the sound

Table 1-14

Attenuation figures for some building materials in the range 100 to 3200 Hz

Structural component	Attenuation dB	Structural component	Attenuation dB
Brickwork rendered, 12 cm thick	45	Single door without extra sealing	to 20
Brickwork rendered, 25 cm thick	50	Single door with good seal	30
Concrete wall, 10 cm thick	42	Double door without seal	30
Concrete wall, 20 cm thick	48	Double door with extra sealing	40
Wood wool mat, 8 cm thick	50	Single window without sealing	15
Straw mat, 5 cm thick	38	Spaced double window with seal	30

The reduction in level ΔL obtainable in a room by means of sound-absorbing materials or structures is:

$$\Delta L = 10 \lg \frac{A_2}{A_1} = 10 \lg \frac{T_1}{T_2} \text{ dB}$$

In the formula:

$$A = 0.163 \frac{V}{T} \text{ in m}^2$$

V = volume of room in m^3

T = reverberation time in s in which the sound level L falls by 60 dB after sound emission ceases.

Index 1 relates to the state of the untreated room, Index 2 to a room treated with noise-reduction measures.

1.2.7 Technical values of solids, liquids and gases

Table 1-15

Technical values of solids

Material	Density ρ kg/dm^3	Melting or freezing point °C	Boiling point °C	Linear thermal expansion α mm/K x 10^{-6} 1)	Thermal conductivity λ at 20 °C W/(m · K)	Mean spec. heat c at 0 . . 100 °C J/(kg · K)	Specific electrical resistance ρ at 20 °C Ω mm²/m	Temperature coefficient α of electrical resistance at 20 °C 1/K
E-aluminium F9	2.70	658	2270	23.8	220	920	0.02874	0.0042
Alu alloy AlMgSi 1 F20	2.70	≈ 645		23	190	920	0.0407	0.0036
Lead	11.34	327	1 730	28	34	130	0.21	0.0043
Bronze CuSnPb	8.6 . . 9	≈ 900		≈ 17.5	42	360	≈ 0.027	0.004
Cadmium	8.64	321	767	31.6	92	234	0.762	0.0042
Chromium	6.92	1 800	2 400	8.5		452	0.028	
Iron, pure	7.88	1 530	2 500	12.3	71	464	0.10	0.0058
Iron, steel	≈ 7.8	≈ 1 350		≈ 11.5	46	485	0.25 . . 0.10	≈ 0.005
Iron, cast	≈ 7.25	≈ 1 200		≈ 11	46	540	0.6 . . 1	0.0045
Gold	19.29	1 063	2 700	14.2	309	130	0.022	0 0038
Constantan Cu + Ni	8 . . 8.9	1 600		16.8	22	410	0.48 . . 0.50	≈ 0.00005
Carbon diamond	3.51	≈ 3 600	4 200	1.3		502		
Carbon graphite	2.25			7.86	5	711		
E-copper F30	8.92	1 083	2 330	16.5	385	393	0.01786	0.00392
E-copper F20	8.92	1 083	2 330	16.5	385	393	0.01754	0.00392
Magnesium	1.74	650	1110	25.0	167	1034	0.0455	0.004

1) between 0 °C and 100 °C

(continued)

Table 1-15 (continued)

Technical values of solids

Material	Density ρ kg/dm³	Melting or freezing point °C	Boiling point °C	Linear thermal expansion α mm/K x 10^{-6} [1]	Thermal conductivity λ at 20 °C W/(m · K)	Mean spec. heat c at 0 . . 100 °C J/(kg · K)	Specific electrical resistance ρ at 20 °C Ω mm²/m	Temperature coefficient α of electrical resistance at 20 °C 1/K
Brass (Ms 58)	8.5	912		17	110	397	≈ 0.0555	0.0024
Nickel	8.9	1 455	3 000	13	83	452	≈ 0.12	0.0046
Platinum	21.45	1 773	3 800	8.99	71	134	≈ 0.11	0.0039
Mercury	13.546	38.83	357	61	8.3	139	0.698	0.0008
Sulphur (rhombic)	2.07	113	445	90	0.2	720		
Selenium (metallic)	4.26	220	688	66		351		
Silver	10.50	960	1 950	19.5	421	233	0.0165	0.0036
Tungsten	19.3	3 380	6 000	4.50	167	134	0.06	0.0046
Zinc	7.23	419	907	16.50	121	387	0.0645	0.0037
Tin	7.28	232	2 300	26.7	67	230	0.119	0.004

[1] between 0 °C and 100 °C

Table 1-16

Technical values of liquids

Material	Chemical formula	Density ρ kg/dm³	Melting or freezing point °C	Boiling point at 760 Torr °C	Expansion coefficient x 10^{-3} at 18 °C	Thermal conductivity λ at 20 °C W/(m · K)	Specific heat c_p at 0 °C J/(kg · K)	Relative dielectric constant ε_r at 180 °C
Acetone	C_3H_6O	0.791	— 95	56.3	1.43		2 160	21.5
Ethyl alcohol	C_2H_6O	0.789	— 114	78.0	1.10	0.2	2 554	25.8
Ethyl ether	$C_4H_{10}O$	0.713	— 124	35.0	1.62	0.14	2 328	4.3
Ammonia	NH_3	0.771	— 77.8	— 33.5		0.022	4 187	14.9
Aniline	C_6H_7N	1.022	— 6.2	184.4	0.84		2 064	7.0
Benzole	C_6H_6	0.879	+ 5.5	80.1	1.16	0.14	1 758	2.24
Acetic acid	$C_2H_4O_2$	1.049	+ 16.65	117.8	1.07		2 030	6.29
Glycerine	$C_3H_8O_3$	1.26	— 20	290	0.50	0.29	2 428	56.2
Linseed oil		0.94	— 20	316		0.15		2.2
Methyl alcohol	CH_4O	0.793	— 97.1	64.7	1.19	0.21	2 595	31.2
Petroleum		0.80			0.99	0.16	2 093	2.1
Castor oil		0.97			0.69		1 926	4.6
Sulphuric acid	H_2SO_4	1.834	— 10.5	338	0.57	0.46	1 385	> 84
Turpentine	$C_{10}H_{16}$	0.855	— 10	161	9.7	0.1	1 800	2.3
Water	H_2O	1.00[1)]	0	106	0.18	0.58	4 187	88

1) at 4 °C

Table 1- 17

Technical values of gases

Material	Chemical formula	Density ρ[1) kg/m^3	Melting point °C	Boiling point °C	Thermal conductivity λ 10^{-2} W/(m · K)	Specific heat c_p at 0 °C J/(kg · K)	Relative[1) dielectric constant ε_r
Ammonia	NH_3	0.771	— 77.7	— 33.4	2.17	2 060	1.0072
Ethylene	C_2H_4	1.260	— 169.4	— 103.5	1.67	1 611	1.001456
Argon	Ar	1.784	— 189.3	— 185.9	1.75	523	1.00056
Acetylene	C_2H_2	1.171	— 81	— 83.6	1.84	1 511	
Butane	C_4H_{10}	2.703	— 135	— 0.5	0.15		
Chlorine	Cl_2	3.220	— 109	— 35.0	0.08	502	1.97
Helium	He	0.178	— 272	— 268.9	1.51	5 233	1.000074
Carbon monoxide	CO	1.250	— 205	— 191.5	0.22	1 042	1.0007
Carbon dioxide	CO_2	1.977	— 56	— 78.5	1.42	819	1.00095
Krypton	Kr	3.743	— 157.2	— 153.2	0.88		
Air	CO_2 free	1.293		— 194.0	2.41	1 004	1.000576
Methane	CH_4	0.717	— 182.5	— 161.7	3.3	2 160	1.000953
Neon	Ne	0.8999	— 248.6	— 246.1	4.6		
Ozone	O_3	2.22	— 252	— 112			
Propane	C_2H_8	2.019	— 189.9	— 42.6			
Oxygen	O_2	1.429	— 218.83	— 192.97	2.46	1 038	1.000547
Sulphur hexafluoride	SF_6	6.07[2)	— 50.8[3)	— 63	1.28[2)	670	1.0021[2)
Nitrogen	N_2	1.250	— 210	— 195.81	2.38	1042	1.000606
Hydrogen	H_2	0.0898	— 259.2	— 252.78	17.54	14 235	1.000264

1) at 0 °C and 1013 mbar
2) at 20 °C and 1013 mbar
3) at 2.26 bar

1.3 Strength of materials

1.3.1 Fundamentals and definitions

External forces *F* acting on a cross-section A of a structural element can give rise to tensile stresses (σ_z), compressive stresses (σ_d), bending stresses (σ_b), shear stresses (τ_s) or torsional stresses (τ_t). If a number of stresses are applied simultaneously to a component, i. e. compound stresses, this component must be designed according to the formulae for compound strength. In this case the following rule must be observed:

Normal stresses σ_z. σ_d. σ_b,
Tangential stresses (shear and torsional stresses) τ_s, τ_t.

are to be added arithmetically;

Normal stresses σ_b with shear stresses τ_s,
Normal stresses σ_b with torsional stresses τ_t,

are to be added geometrically.

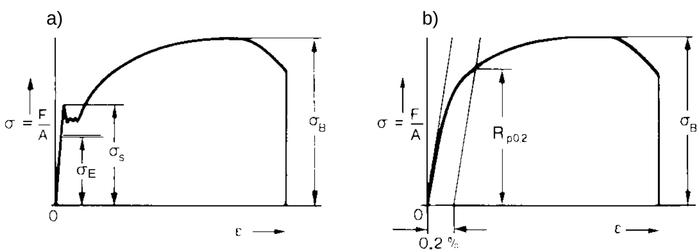


Fig. 1-2

Stress-strain diagram, a) Tensile test with pronounced yield point, material = structural steel; b) Tensile test without pronounced yield point, material = Cu/Al, ε Elongation, σ Tensile stress, σ_s Stress at yield point, σ_E Stress at proportionality limit, R_{p02} Stress with permanent elongation less than 0.2 %, σ_B Breaking stress.

Elongation $\varepsilon = \Delta l / l_0$ (or compression in the case of the compression test) is found from the measured length l_0 of a bar test specimen and its change in length $\Delta l = l - l_0$ in relation to the tensile stress σ_z, applied by an external force F. With stresses below the proportionality limit σ_E elongation increases in direct proportion to the stress σ (Hooke's law).

The ratio $\dfrac{\text{Stress } \sigma}{\text{Elongation } \varepsilon} = \dfrac{\sigma_E}{\varepsilon_E} = E$ is termed the elasticity modulus.

E is an imagined stress serving as a measure of the resistance of a material to deformation due to tensile or compressive stresses; it is valid only for the elastic region.

According to DIN 1602/2 and DIN 50143, *E* is determined in terms of the load $\sigma_{0.01}$, i.e. the stress at which the permanent elongation is 0.01 % of the measured length of the test specimen.

If the stresses exceed the yield point σ_s, materials such as steel undergo permanent elongation. The ultimate strength, or breaking stress, is denoted by σ_B, although a bar does not break until the stress is again being reduced. Breaking stress σ_B is related to the elongation on fracture δ of a test bar. Materials having no marked proportional limit or elastic limit, such as copper and aluminium, are defined in terms of the so-called $R_{p0.2}$-limit, which is that stress at which the permanent elongation is 0.2 % after the external force has been withdrawn, cf. DIN 50144.

For reasons of safety, the maximum permissible stresses, σ_{max} or τ_{max} in the material must be below the proportional limit so that no permanent deformation, such as elongation or deflection, persists in the structural component after the external force ceases to be applied.

Table 1-18

Material	Elasticity modulus *E* N/mm^2 [1)]
Structural steel in general, spring steel (unhardened), cast steel	210 000
Grey cast iron	100 000
Electro copper, Al bronze with 5 % Al, rolled	110 000
Red brass	90 000
E-AlMgSi 0.5	75 000
E-Al	65 000
Magnesium alloy	45 000
Wood	10 000

1) Typical values.

Fatigue strength (endurance limit) is present when the maximum variation of a stress oscillating about a mean stress is applied “infinitely often” to a loaded material (at least 10^7 load reversals in the case of steel) without giving rise to excessive deformation or fracture.

Cyclic stresses can occur in the form of a stress varying between positive and negative values of equal amplitude, or as a stress varying between zero and a certain maximum value. Cyclic loading of the latter kind can occur only in compression or only in tension.

Depending on the manner of loading, fatigue strength can be considered as bending fatigue strength, tension-compression fatigue strength or torsional fatigue strength. Structural elements which have to withstand only a limited number of load reversals can be subjected to correspondingly higher loads. The resulting stress is termed the fatigue limit.

One speaks of creep strength when a steady load with uniform stress is applied, usually at elevated temperatures.

1.3.2 Tensile and compressive strength

If the line of application of a force *F* coincides with the centroidal axis of a prismatic bar of cross section *A* (Fig.1-3), the normal stress uniformly distributed over the cross-section area and acting perpendicular to it is

$$\sigma = \frac{F}{A} .$$

With the maximum permissible stress σ_{max} for a given material and a given loading, the required cross section or the maximum permissible force, is therefore:

$$A = \frac{F}{\sigma_{max}} \text{ or } F = \sigma_{max} \cdot A.$$

Example:

A drawbar is to be stressed with a steady load of $F = 180\ 000$ N.

The chosen material is structural steel St 37 with $\sigma_{max} = 120$ N/mm².

Required cross section of bar:

$$A = \frac{E}{\sigma_{max}} = \frac{180\ 000\ N}{120\ N/mm^2} = 1500\ mm^2.$$

Round bar of d = 45 mm *chosen.*

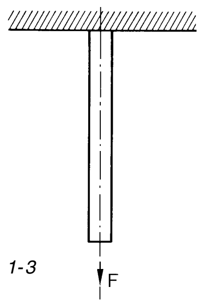


Fig. 1-3

1.3.3 Bending strength

The greatest bending action of an external force, or its greatest bending moment *M,* occurs at the point of fixing *a* in the case of a simple cantilever, and at point *c* in the the case of a centrally loaded beam on two supports.

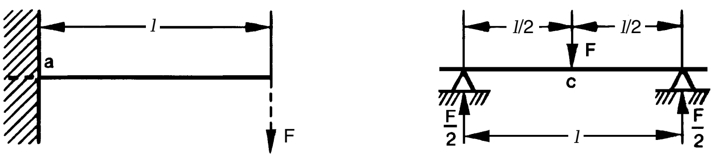


Fig. 1-4

Maximum bending moment at a: M = Fl; at c: M = Fl/4

In position *a* and *c*, assuming the beams to be of constant cross section, the bending stresses σ_b are greatest in the filaments furthermost from the neutral axis. *M* may be greater, the greater is σ_{max} and the "more resistant" is the cross-section. The following cross sections have moments of resistance *W* in cm, if *a, b, h* and *d* are stated in cm.

The maximum permissible bending moment is $M = W \cdot \sigma_{max}$ and the required moment of resistance

$$W = \frac{M}{\sigma_{max}} .$$

Example:

A mild-steel stud $\left(\sigma_{max} = 70\ \text{N/mm}^2\right)$ with an unsupported length of $l = 60$ mm is to be loaded in the middle with a force $F = 30\,000$ N. Required moment of resistance is:

$$W = \frac{M}{\sigma_{max}} = \frac{F \cdot l}{4 \cdot \sigma_{max}} = \frac{30\,000\ \text{N} \cdot 60\ \text{mm}}{4 \cdot 70\ \text{N/mm}^2} = 6.4 \cdot 10^3\ \text{mm}^3.$$

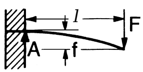
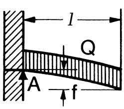
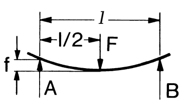
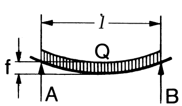
According to Table 1-22, the moment of resistance W with bending is $W \approx 0.1 \cdot d^3$.

The diameter of the stud will be: $d = \sqrt[3]{10\ W}$, $\quad d = \sqrt[3]{64\,000} = \sqrt[3]{64} \cdot 10 = 40$ mm.

1.3.4 Loadings on beams

Table 1-19

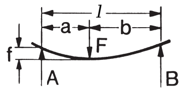
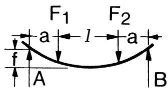
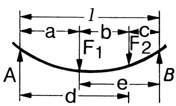
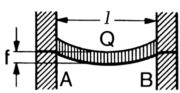
Bending load

Case	Reaction force Bending moment	Required moment of resistance, max. permissible load	Deflection
	$A = F$ $M_{max} = F\,l$	$W = \frac{F\,l}{\sigma_{max}}$ $F = \frac{\sigma_{max}\,W}{l}$	$f = \frac{F\,l^3}{3\,E\,J}$
	$A = Q$ $M_{max} = \frac{Q\,l}{2}$	$W = \frac{Q\,l}{2\,\sigma_{max}}$ $Q = \frac{2\,\sigma_{max}\,W}{l}$	$f = \frac{Q\,l^3}{8\,E\,J}$
	$A = B = \frac{F}{2}$ $M_{max} = \frac{F\,l}{4}$	$W = \frac{F\,l}{4\,\sigma_{max}}$ $F = \frac{4\,\sigma_{max}\,W}{l}$	$f = \frac{F\,l^3}{48\,E\,J}$
	$A = B = \frac{Q}{2}$ $M_{max} = \frac{Q\,l}{8}$	$W = \frac{Q\,l}{8\,\sigma_{max}}$ $Q = \frac{8\,\sigma_{max}\,W}{l}$	$f = \frac{5}{384} \cdot \frac{Q\,l^3}{E\,J}$

(continued)

Table 1-19 (continued)

Bending load

Case	Reaction force Bending moment	Required moment of resistance, max. permissible load	Deflection
	$A = \frac{F\,b}{l}$ $B = \frac{F\,a}{l}$ $M_{max} = A\,a = B\,b$	$W = \frac{F\,a\,b}{l\,\sigma_{max}}$ $F = \frac{\sigma_{max}\,W\,l}{a\,b}$	$f = \frac{F\,a^2\,b^2}{3\,E\,J\,l}$
	*for* $F_1 = F_2 = F$ [1)] $A = B = F$ $M_{max} = F\,a$	$W = \frac{F\,a}{\sigma_{max}}$ $F = \frac{\sigma_{max}\,W}{a}$	$f = \frac{F\,a}{24\,E\,J}$ $[3\,(l + 2\,a)^2 - 4\,a^2]$
	$A = \frac{F_1\,e + F_2\,c}{l}$ $B = \frac{F_1\,a + F_2\,d}{l}$	$W_1 = \frac{A\,a}{\sigma_{max}}$ $W_2 = \frac{B\,c}{\sigma_{max}}$ Determine beam for greatest *"W"*	$f = \frac{F_1\,a^2\,e^2 + F_2 l^2\,d^2}{3\,E\,J\,l}$
	$A = B = \frac{Q}{l}$ $M_{max} = \frac{Q\,l}{12}$	$W = \frac{Q\,l}{12\,\sigma_{zul}}$ $Q = \frac{12\,\sigma_{zul}\,W}{l}$	$f = \frac{Q}{E\,J} \cdot \frac{l^3}{384}$

A and *B* = Section at risk.

F = Single point load, *Q* = Uniformly distributed load.

1) If F_1 und F_2 are not equal, calculate with the third diagram.

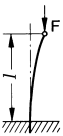
1.3.5 Buckling strength

Thin bars loaded in compression are liable to buckle. Such bars must be checked both for compression and for buckling strength, cf. DIN 4114.

Buckling strength is calculated with Euler's formula, a distinction being drawn between four cases.

Table 1-20

Buckling

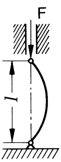


Case I

One end fixed, other end free

$$F = \frac{10\,E\,J}{4\,s\,l^2}$$

$$J = \frac{4\,s\,F\,l^2}{10\,E}$$

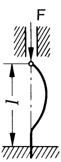


Case II

Both ends free to move along bar axis

$$F = \frac{10\,E\,J}{s\,l^2}$$

$$J = \frac{s\,F\,l^2}{10\,E}$$

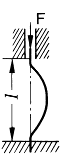


Case III

One end fixed, other end free to move along bar axis

$$F = \frac{20\,E\,J}{s\,l^2}$$

$$J = \frac{s\,F\,l^2}{20\,E}$$



Case IV

Both ends fixed, movement along bar axis

$$F = \frac{40\,E\,J}{s\,l^2}$$

$$J = \frac{s\,F\,l^2}{40\,E}$$

E = Elasticity modulus of material
J = Minimum axial moment of inertia
F = Maximum permissible force
l = Length of bar

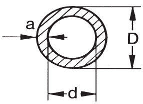
s = Factor of safety:
for cast iron = 8,
for mild carbon steel = 5,
for wood = 10.

1.3.6 Maximum permissible buckling and tensile stress for tubular rods

Threaded steel tube (gas pipe) DIN 2440, Table 1[1)]
or seamless steel tube DIN 2448[2)].

$$F_{buck} = \frac{10\,E}{s\,l^2} \cdot J = \frac{10\,E}{s\,l^2} \cdot \frac{D^4 - d^4}{20} \quad \text{where } J \approx \frac{D^4 - d^4}{20} \text{ from Table 1-22}$$

$$F_{ten} = A \cdot \sigma_{max}$$



in which
F Force
E Elasticity modulus = 210 000 N/mm²
J Moment of inertia in cm⁴
s Factor of safety = 5
σ_{max} Max. permissible stress
A Cross-section area
D Outside diameter
d Inside diameter
l Length

Fig. 1-5

Table 1-21

Nominal diameter	Dimensions D inch	D mm	a mm	Cross-sections A mm²	Moment of inertia J cm⁴	Weight of tube kg/m	F_{buck} for tube length $l \approx$ 0.5 m N	1 m N	1.5 m N	2 m N	2.5 m N	3 m N	F_{ten} N
10	⅜	17.2	2.35	109.6	0.32	0.85	5400	1350	600	340	220	150	6600
15	½	21.3	2.65	155.3	0.70	1.22	11800	2950	1310	740	470	330	9300
20	¾	26.9	2.65	201.9	1.53	1.58	25700	6420	2850	1610	1030	710	12100
25	1	33.7	3.25	310.9	3.71	2.44	62300	15600	6920	3900	2490	1730	18650
	0.8	25	2	144.5	0.98	1.13	16500	4100	1830	1030	660	460	17350
	0.104	31.8	2.6	238.5	2.61	1.88	43900	11000	4880	2740	1760	1220	28600

1) No test values specified for steel ST 00.
2) σ_{max} = 350 N/mm² for steel ST 35 DIN 1629 seamless steel tube, cf. max. permissible buckling stress for structural steel, DIN 1050 Table 3.

1.3.7 Shear strength[1)]

Two equal and opposite forces F acting perpendicular to the axis of a bar stress this section of the bar in shear. The stress is

$$\tau_s = \frac{F}{A} \text{ or for given values of } F \text{ and } \tau_{s\,max}, \text{ the required cross section is}$$

$$A = \frac{F}{\tau_{s\,max}}$$

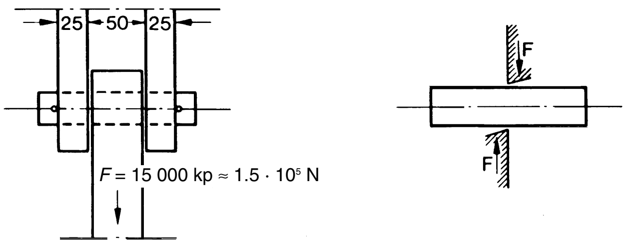


Fig. 1-6

Pull-rod coupling

Stresses in shear are always combined with a bending stress, and therefore the bending stress σ_b has to be calculated subsequently in accordance with the following example.

Rivets, short bolts and the like need only be calculated for shear stress.

Example:

Calculate the cross section of a shackle pin of structural steel ST 50-1[2)], with $R_{p\,0.2\,min}$ = 300 N/mm² and $\tau_{s\,max}$ = 0.8 $R_{p\,0.2\,min}$, for the pull-rod coupling shown in Fig. 1-6.

1. Calculation for shear force:

$$A = \frac{F}{2\,\tau_{s\,max}} = \frac{150\,000 \text{ N}}{2 \cdot (0.8 \cdot 300) \text{ N/mm}^2} = 312 \text{ mm}^2$$

yields a pin diameter of $d \approx$ 20 mm, with W = 0.8 · 10³ mm³ (from $W \approx 0.1 \cdot d^3$, see Table 1-22).

1) For maximum permissible stresses on steel structural components of transmission towers and structures for outdoor switchgear installations, see VDE 0210.
2) Yield point of steel ST 50-1 $\sigma_{0.2\,min}$ = 300 N/mm², DIN 17100 Table 1 (Fe 50-1).

2. Verification of bending stress:

The bending moment for the pin if $F\,l/4$ with a singlepoint load, and $F\,l/8$ for a uniformly distributed load. The average value is

$$M_b = \frac{\frac{F\,l}{4} + \frac{F\,l}{8}}{2} = \frac{3}{16}\,F\,l$$

when F = $1.5 \cdot 10^5$ N, l = 75 mm becomes:

$$M_b = \frac{3}{16} \cdot 1.5 \cdot 10^5\ \text{N} \cdot 75\ \text{mm} \approx 21 \cdot 10^5\ \text{N} \cdot \text{mm};$$

$$\sigma_B = \frac{M_b}{W} = \frac{21 \cdot 10^5\ \text{N} \cdot \text{mm}}{0.8 \cdot 10^3\ \text{mm}^3} \approx 262 \cdot 10^3\ \frac{\text{N}}{\text{mm}^2} = 2.6 \cdot 10^5\ \frac{\text{N}}{\text{mm}^2}$$

i. e. a pin calculated in terms of shear with d = 20 mm will be too weak. The required pin diameter d calculated in terms of bending is

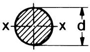
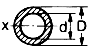
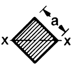
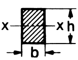
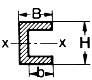
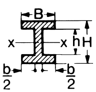
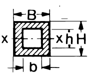
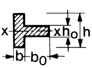
$$W = \frac{M_b}{\sigma_{max}} = \frac{21 \cdot 10^5\ \text{N} \cdot \text{mm}}{300\ \text{N/mm}^2} = 7 \cdot 10^3\ \text{mm}^2 = 0.7\ \text{cm}^3$$

$$d \approx \sqrt[3]{10 \cdot W} = \sqrt[3]{10 \cdot 7 \cdot 10^3\ \text{mm}^3} = \sqrt[3]{70} = 41.4\ \text{mm} \approx 42\ \text{mm}.$$

i. e. in view of the bending stress, the pin must have a diameter of 42 mm instead of 20 mm.

1.3.8 Moments of resistance and moments of inertia

Table 1-22

Cross-section	Moment of resistance torsion W[4)] cm^3	bending[1)] W[4)] cm^3	Moment of inertia polar[1)] J_p cm^4	axial[2)] J cm^4
	$0.196\ d^3$ $\approx 0.2\ d^3$	$0.098\ d^3$ $\approx 0.1\ d^3$	$0.098\ d^4$ $\approx\ 0.1\ d^4$	$0.049\ d^4$ $\approx 0.05\ d^4$
	$0.196\ \frac{D^4 - d^4}{D}$	$0.098\ \frac{D^4 - d^4}{D}$	$0.098\ (D^4 - d^4)$	$0.049\ (D^4 - d^4)$ $\approx \frac{D^4 - d^4}{20}$
	$0.208\ a^3$	$0.018\ a^3$	$0.167\ a^4$	$0.083\ a^4$
	$0.208\ k\ b^2\ h$ [3)]	$\frac{b\ h^2}{6} = 0.167\ b\ h^2$	$\frac{b\ h}{12}\ (b^2 + h^2)$	$\frac{b\ h^3}{12} = 0.083\ b\ h^3$
		$\frac{B\ H^3 - b\ h^3}{6\ H}$		$\frac{B\ H^3 - b\ h^3}{12}$
		$\frac{B\ H^3 - b\ h^3}{6\ H}$		$\frac{B\ H^3 - b\ h^3}{12}$
		$\frac{B\ H^3 - b\ h^3}{6\ H}$		$\frac{B\ H^3 - b\ h^3}{12}$
		$\frac{b\ h^3 + b_o\ h_o^3}{6\ h}$		$\frac{b\ h^3 + b_o\ h_o^3}{12}$

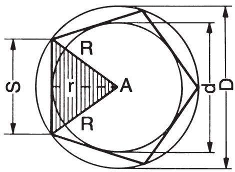
1) Referred to CG of area.
2) Referred to plotted axis.
3) Values for k:

if $h : b$ =	1	1.5	2	3	4
then k =	1	1.11	1.18	1.27	1.36

4) Symbol Z is also applicable, see DIN VDE 0103

1.4 Geometry, calculation of areas and solid bodies

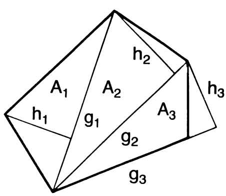
1.4.1 Area of polygons



Regular polygons (n angles)
The area A, length of sides S and radii of the outer and inner circles can be taken from Table 1-23 below.

Table 1-23

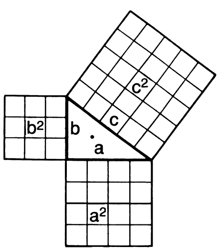
Number of sides	Area A			Side S		Outer radius		Inner radius	
						R		r	
n	$S^2 \times$	$R^2 \times$	$r^2 \times$	$R \times$	$r \times$	$S \times$	$r \times$	$R \times$	$S \times$
3	0.4330	1.2990	5.1962	1.7321	3.4641	0.5774	2.0000	0.5000	0.2887
4	1.0000	2.0000	4.0000	1.4142	2.0000	0.7071	1.4142	0.7071	0.5000
5	1.7205	2.3776	3.6327	1.1756	1.4531	0.8507	1.2361	0.8090	0.6882
6	2.5981	2.5981	3.4641	1.0000	1.1547	1.0000	1.1547	0.8660	0.8660
8	4.8284	2.8284	3.3137	0.7654	0.8284	1.3066	1.0824	0.9239	1.2071
10	7.6942	2.9389	3.2492	0.6180	0.6498	1.6180	1.0515	0.9511	1.5388
12	11.196	3.0000	3.2154	0.5176	0.5359	1.9319	1.0353	0.9659	1.8660



Irregular polygons

$$A = \frac{g_1\,h_1}{2} + \frac{g_2\,h_2}{2} + \ldots$$

$$= \frac{1}{2}\,(g_1\,h_1 + g_2\,h_2 + \ldots)$$



Pythagoras theorem

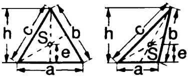
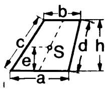
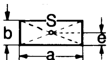
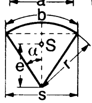
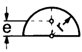
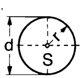
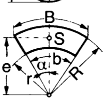
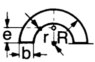
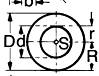
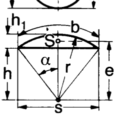
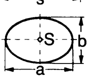
$c^2 = a^2 + b^2$; $\quad c = \sqrt{a^2 + b^2}$

$a^2 = c^2 - b^2$; $\quad a = \sqrt{c^2 - b^2}$

$b^2 = c^2 - a^2$; $\quad b = \sqrt{c^2 - a^2}$

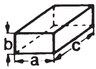
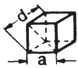
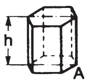
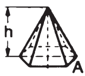
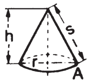
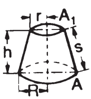
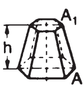
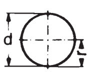
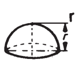
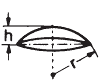
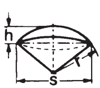
1.4.2 Areas and centres of gravity

Table 1-24

Shape of surface		A = area	U = perimeter S = centre of gravity (cg) e = distance of cg
Triangle		$A = \frac{1}{2} a h$	$U = a + b + c$ $e = \frac{1}{3} h$
Trapezium		$A = \frac{a + b}{2} \cdot h$	$U = a + b + c + d$ $e = \frac{h}{3} \cdot \frac{a + 2b}{a + b}$
Rectangle		$A = a b$	$U = 2 (a + b)$
Circle segment		$A = \frac{b r}{2} = \frac{\alpha^0}{180} r \pi$ $b = r \pi \frac{\alpha^0}{180}$	$U = 2 r + b$ $e = \frac{2}{3} r \frac{\sin \alpha}{\alpha^0} \cdot \frac{180}{\pi}$
Semicircle		$A = \frac{1}{2} \pi r^2$	$U = r (2 + \pi) = 5.14 r$ $e = \frac{1}{3} \cdot \frac{r}{\pi} = 0.425 r$
Circle		$A = r^2 \pi = \pi \frac{d^2}{4}$	$U = 2 \pi r = \pi d$
Annular segment		$A = \frac{\pi}{180} \alpha^0 (R^2 - r^2)$	$U = 2 (R - r) + B + b$ $e = \frac{2}{3} \cdot \frac{R^2 - r^2}{R^2 - r^2} \cdot \frac{\sin \alpha}{\alpha^0} \cdot \frac{180}{\pi}$
Semi-annulus		$A = \frac{\pi}{2} \alpha^0 (R^2 - r^2)$	if $b < 0.2 R$, then $e \approx 0.32 (R + r)$
Annulus		$A = \pi (R^2 - r^2)$	$U = 2 \pi (R + r)$
Circular segment		$A = \frac{\alpha^0}{180} r^2 \pi - \frac{s h}{2}$ $s = 2 \sqrt{r^2 - h^2}$	$U = 2 \sqrt{r^2 - h^2} + \frac{\pi r \alpha^0}{90}$ $e = \frac{s^2}{12 \cdot A}$
Ellipse		$A = \frac{a b}{4} \pi$	$U = \frac{\pi}{2} \left[1.5 (a + b) - \sqrt{ab} \right]$

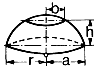
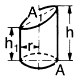
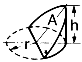
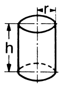
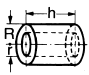
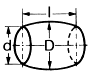
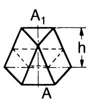
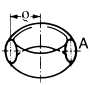
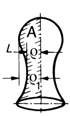
1.4.3 Volumes and surface areas of solid bodies

Table 1-25

Shape of body	V = volume	O = Surface A = Area
Solid rectangle 	$V = a\,b\,c$	$O = 2\,(a\,b + a\,c + b\,c)$
Cube 	$V = a^3 = \dfrac{d^3}{2.828}$	$O = 6\,a^2 = 3\,d^2$
Prism 	$V = A\,h$	$O = U\,h + 2\,A$ A = base surface
Pyramid 	$V = \dfrac{1}{3}\,A\,h$	O = A + Nappe
Cone 	$V = \dfrac{1}{3}\,A\,h$	$O = \pi\,r\,s + \pi\,r^2$ $s = \sqrt{h^2 + r^2}$
Truncated cone 	$V = (R^2 + r^2 + R\,r) \cdot \dfrac{\pi\,h}{3}$	$O = (R + r)\,\pi\,s + \pi\,(R^2 + r^2)$ $s = \sqrt{h^2 + (R - r)^2}$
Truncated pyramid 	$V = \dfrac{1}{3}\,h\,(A + A_1 + \sqrt{AA_1})$	$O = A + A_1$ + Nappe
Sphere 	$V = \dfrac{4}{3}\,\pi\,r^3$	$O = 4\,\pi\,r^2$
Hemisphere 	$V = \dfrac{2}{3}\,\pi\,r^3$	$O = 3\,\pi\,r^2$
Spherical segment 	$V = \pi\,h^2 \left(r - \dfrac{1}{3}\,h\right)$	$O = 2\,\pi\,r\,h + \pi\,(2\,r\,h - h^2) = \pi\,h\,(4\,r - h)$
Spherical sector 	$V = \dfrac{2}{3}\,\pi\,r^2\,h$	$O = \dfrac{\pi\,r}{2}\,(4\,h + s)$

(continued)

Table 1-25 (continued)

Shape of body		V = Volume	O = Surface A = Area
Zone of sphere		$V = \frac{\pi h}{3}(3a^2 + 3b^2 + h^2)$	$O = \pi (2rh + a^2 + b^2)$
Obliquely cut cylinder		$V = \pi r^2 \frac{h + h_1}{2}$	$O = \pi r(h + h_1) + A + A_1$
Cylindrical wedge		$V = \frac{2}{3} r^2 h$	$0 = 2rh + \frac{\pi}{2} r^2 + A$
Cylinder		$V = \pi r^2 h$	$O = 2\pi rh + 2\pi r^2$
Hollow cylinder		$V = \pi h (R^2 - r^2)$	$O = 2\pi h(R + r) + 2\pi (R^2 - r^2)$
Barrel		$V = \frac{\pi}{15} l \cdot$ $(2D^2 + Dd + 0.75d^2)$	$O = \frac{D + d}{2}\pi d + \frac{\pi}{2} d^2$ (approximate)
Frustum		$V = \left(\frac{A - A_1}{2} + A_1\right) h$	$O = A + A_1$ + areas of sides
Body of rotation (ring)		$V = 2\pi \varrho A$ A = cross-section	O = circumference of cross-section x $2\pi\varrho$
Pappus' theorem for bodies of revolution		Volume of turned surface (hatched) x path of its centre of gravity $V = A\,2\pi\varrho$	Length of turned line x path of its centre of gravity $O = L\,2\pi\varrho_1$

2 General Electrotechnical Formulae

2.1 Electrotechnical symbols as per DIN 1304 Part 1

Table 2-1

Mathematical symbols for electrical quantities (general)

Symbol	Quantity	SI unit
Q	quantity of electricity, electric charge	C
E	electric field strength	V/m
D	electric flux density, electric displacement	C/m^2
U	electric potential difference	V
φ	electric potential	V
ε	permittivity, dielectric constant	F/m
ε_o	electric field constant, $\varepsilon_o = 0.885419 \cdot 10^{-11}$ F/m	F/m
ε_r	relative permittivity	1
C	electric capacitance	F
I	electric current	A
J	electric current density	A/m^2
x, γ, σ	specific electric conductivity	S/m
ρ	specific electric resistance	Ωm
G	electric conductance	S
R	electric resistance	Ω
θ	electromotive force	A

Table 2-2

Mathematical symbols for magnetic quantities (general)

Symbol	Quantity -	SI unit
Φ	magnetic flux	Wb
B	magnetic induction	T
H	magnetic field strength	A/m
V	magnetomotive force	A
φ	magnetic potential	A
μ	permeability	H/m
μ_o	absolute permeability, $\mu_o = 4\ \pi \cdot 10^{-7} \cdot$ H/m	H/m
μ_r	relative permeability	1
L	inductance	H
L_{mn}	mutual inductance	H

Table 2-3

Mathematical symbols for alternating-current quantities and network quantities

Symbol	Quantity	SI unit
S	apparent power	W, VA
P	active power	W
Q	reactive power	W, Var
D	distortion power	W
φ	phase displacement	rad
ϑ	load angle	rad
λ	power factor, $\lambda = P/S$, λ cos φ [1)]	1
δ	loss angle	rad
d	loss factor, $d = \tan \delta$	1
Z	impedance	Ω
Y	admittance	S
R	resistance	Ω
G	conductance	S
X	reactance	Ω
B	susceptance	S
γ	impedance angle, $\gamma = \arctan X/R$	rad

Table 2-4

Numerical and proportional relationships

Symbol	Quantity	SI unit
η	efficiency	1
s	slip	1
p	number of pole-pairs	1
w, N	number of turns	1
$ü$	transformation ratio	1
m	number of phases and conductors	1
γ	amplitude factor	1
k	overvoltage factor	1
ν	ordinal number of a periodic component	1
s	wave content	1
g	fundamental wave content	1
k	harmonic content, distortion factor	1
ζ	increase in resistance due to skin effect, $\zeta = R_{\sim} / R_{-}$	1

1) Valid only for sinusoidal voltage and current.

2.2 Alternating-current quantities

With an alternating current, the instantaneous value of the current changes its direction as a function of time i = f(t). If this process takes place periodically with a period of duration T, this is a periodic alternating current. If the variation of the current with respect to time is then sinusoidal, one speaks of a sinusoidal alternating current.

The frequency f and the angular frequency ω are calculated from the periodic time T with

$$f = \frac{1}{T} \text{ and } \omega = 2\,\pi\, f = \frac{2\pi}{T}.$$

The *equivalent d. c. value* of an alternating current is the average, taken over one period, of the value:

$$|\bar{i}| = \frac{1}{T}\int_0^T |i|\,\mathrm{d}t = \frac{1}{2\pi}\int_0^{2\pi} |i|\,\mathrm{d}\omega t.$$

This occurs in rectifier circuits and is indicated by a moving-coil instrument, for example.

The root-mean-square value (rms value) of an alternating current is the square root of the average of the square of the value of the function with respect to time.

$$I = \sqrt{\frac{1}{T}\cdot\int_0^T i^2\mathrm{d}t} = \sqrt{\frac{1}{2\pi}\cdot\int_0^{2\pi} i^2\mathrm{d}\omega t}.$$

As regards the generation of heat, the root-mean-square value of the current in a resistance achieves the same effect as a direct current of the same magnitude.

The root-mean-square value can be measured not only with moving-coil instruments, but also with hot-wire instruments, thermal converters and electrostatic voltmeters.

A non-sinusoidal current can be resolved into the fundamental oscillation with the fundamental frequency f and into harmonics having whole-numbered multiples of the fundamental frequency. If I_1 is the rms value of the fundamental oscillation of an alternating current, and I_2, I_3 etc. are the rms values of the harmonics having frequencies $2\,f$, $3\,f$, etc., the rms value of the alternating current is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots}$$

If the alternating current also includes a direct-current component i_{-}, this is termed an undulatory current. The rms value of the undulatory current is

$$I = \sqrt{I_{-}^2 + I_1^2 + I_2^2 + I_3^2 + \ldots}$$

The fundamental oscillation content g is the ratio of the rms value of the fundamental oscillation to the rms value of the alternating current

$$g = \frac{I_1}{I}.$$

The harmonic content k (distortion factor) is the ratio of the rms value of the harmonics to the rms value of the alternating current.

$$k = \frac{\sqrt{I_2^2 + I_3^2 + \ldots}}{I} = \sqrt{1 - g^2}$$

The fundamental oscillation content and the harmonic content cannot exceed 1.

In the case of a sinusoidal oscillation

the fundamental oscillation content $g = 1$,
the harmonic content $k = 0$.

Forms of power in an alternating-current circuit

The following terms and definitions are in accordance with DIN 40110 for the sinusoidal wave-forms of voltage and current in an alternating-current circuit.

apparent power	$S = UI = \sqrt{P^2 + Q^2}$,
active power	$P = UI \cdot \cos\varphi = S \cdot \cos\varphi$,
reactive power	$Q = UI \cdot \sin\varphi = S \cdot \sin\varphi$,
power factor	$\cos\varphi = \frac{P}{S}$,
reactive factor	$\sin\varphi = \frac{Q}{S}$.

When a three-phase system is loaded symmetrically, the apparent power is

$$S = 3\,U_1 I_1 = \sqrt{3} \cdot U \cdot I_1,$$

where I_1 is the rms phase current, U_1 the rms value of the phase to neutral voltage and U the rms value of the phase to phase voltage. Also

active power	$P = 3\,U_1 I_1 \cos\varphi = \sqrt{3} \cdot U \cdot I_1 \cdot \cos\varphi$,
reactive power	$Q = 3\,U_1 I_1 \sin\varphi = \sqrt{3} \cdot U \cdot I_1 \cdot \sin\varphi$.

The unit for all forms of power is the watt (W). The unit watt is also termed volt-ampere (symbol VA) when stating electric apparent power, and Var (symbol var) when stating electric reactive power.

Resistances and conductances in an alternating-current circuit

impedance	$Z = \frac{U}{I} = \frac{S}{I^2} = \sqrt{R^2 + X^2}$
resistance	$R = \frac{U\cos\varphi}{I} = \frac{P}{I^2} = Z\cos\varphi = \sqrt{Z^2 - X^2}$
reactance	$X = \frac{U\sin\varphi}{I} = \frac{Q}{I^2} = Z\sin\varphi = \sqrt{Z^2 - R^2}$
inductive reactance	$X_i = \omega L$
capacitive reactance	$X_c = \frac{1}{\omega C}$
admittance	$Y = \frac{I}{U} = \frac{S}{U^2} = \sqrt{G^2 + B^2} = \frac{1}{Z}$
conductance	$G = \frac{I\cos\varphi}{U} = \frac{P}{U^2} = Y\cos\varphi = \sqrt{Y^2 - B^2} = \frac{R}{Z^2}$
conductance	$B = \frac{I\sin\varphi}{U} = \frac{Q}{U^2} = Y\sin\varphi = \sqrt{Y^2 - G^2} = \frac{X}{Z^2}$
inductive susceptance	$B_i = \frac{1}{\omega L}$
capacitive susceptance	$B_c = \omega C$

$\omega = 2\,\pi\,f$ is the angular frequency and φ the phase displacement angle of the voltage with respect to the current. U, I and Z are the numerical values of the alternating-current quantities $\underline{U}$, $\underline{I}$ and $\underline{Z}$.

Complex presentation of sinusoidal time-dependent a. c. quantities

Expressed in terms of the load vector system:

$\underline{U} = \underline{I} \cdot \underline{Z},\ \underline{I} = \underline{U} \cdot \underline{Y}$
The symbols are underlined to denote that they are complex quantities (DIN 1304).

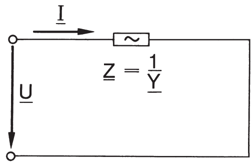


Fig. 2- 1
Equivalent circuit diagram

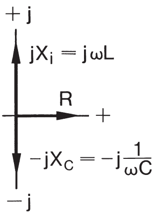


Fig. 2-2
Vector diagram of resistances

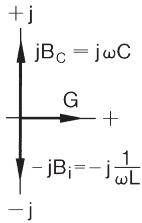


Fig. 2-3
Vector diagram of conductances

If the voltage vector $\underline{U}$ is laid on the real reference axis of the plane of complex numbers, for the equivalent circuit in Fig. 2-1 with $\underline{Z} = R + \mathrm{j}\,X_i$: we have

$$\underline{U} = U,$$

$$\underline{I} = I_w - \mathrm{j}\,I_b = I\,(\cos\varphi - \mathrm{j}\sin\varphi),$$

$$I_w = \frac{P}{U};\ I_b = \frac{Q}{U};$$

$$\underline{S}^{1)} = U\,\underline{I}^* = U\,I\,(\cos\varphi + \mathrm{j}\sin\varphi) = P + \mathrm{j}\,Q,$$

$$S = |\underline{S}| = U\,I = \sqrt{P^2 + Q^2},$$

$$\underline{Z} = R + \mathrm{j}\,X_i = \frac{U}{\underline{I}} = \frac{U}{I\,(\cos\varphi - \mathrm{j}\sin\varphi)} = \frac{U}{I}\,(\cos\varphi + \mathrm{j}\sin\varphi),$$

where $R = \frac{U}{I}\cos\varphi$ and $X_i = \frac{U}{I}\sin\varphi$,

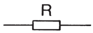
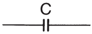
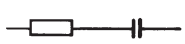
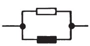
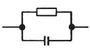
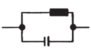
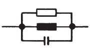
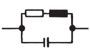
$$\underline{Y} = \mathrm{G} - \mathrm{j}B = \frac{\underline{I}}{U} = \frac{I}{U}\,(\cos\varphi - \mathrm{j}\sin\varphi)$$

where $\mathrm{G} = \frac{I}{U}\cos\varphi$ and $B_i = \frac{I}{U}\sin\varphi$.

[1] $\underline{S}$: See DIN 40110
$\underline{I}^*$ = conjugated complex current vector

Table 2-5

Alternating-current quantities of basic circuits

	Circuit	$\underline{Z}$	$\lvert\underline{Z}\rvert$
1.		R	R
2.		$j\,\omega L$	ωL
3.		$-j/(\omega C)$	$1/\omega C$
4.		$R + j\,\omega L$ [1]	$\sqrt{R^2 + (\omega L)^2}$
5.		$R - j/(\omega C)$	$\sqrt{R^2 + 1/(\omega C)^2}$
6.		$j\,(\omega L - 1/(\omega C))$ [2]	$\sqrt{(\omega L - 1/(\omega C))^2}$
7.		$R + j(\omega L - 1/(\omega C))$ [2]	$\sqrt{R^2 + (\omega L - 1/(\omega C))^2}$
8.		$\dfrac{R\,\omega L}{\omega L - jR}$	$\dfrac{R\,\omega L}{\sqrt{R^2 + (\omega L)^2}}$
9.		$\dfrac{R - j\,\omega C R^2}{1 + (\omega C)^2 R^2}$ [3]	$\dfrac{R}{\sqrt{1 + (\omega C)^2 R^2}}$
10.		$\dfrac{j}{1/(\omega L) - \omega C}$	$\dfrac{1}{\sqrt{(1/\omega L)^2 - (\omega C)^2}}$
11.	 [4]	$\dfrac{1}{1/R + j\,(\omega C - 1/(\omega L))}$ $[\underline{Y} = 1/R^2 + j\,(\omega C - 1/(\omega L))]$	$\dfrac{1}{\sqrt{1/R^2 + (\omega C - 1/(\omega L))^2}}$
12.	 [5]	$\dfrac{R + j\,(L\,(1 - \omega^2 LC) - R^2 C)}{(1 - \omega^2 LC)^2 + (R\,\omega C)^2}$	$\dfrac{\sqrt{R^2 + [L\,(1 - \omega^2 LC) - R^2 C]^2}}{(1 - \omega^2 LC)^2 + (R\,\omega C)^2}$

[1] With small loss angle δ (= $1/\varphi$) $\approx$ tan δ (error at 4° about 1 ‰): $\underline{Z} \approx \omega L\,(\delta + j)$.

[2] Series resonance (voltage resonance) for $\omega L = 1/(\omega C)$:

$X_{res} = |X_L| = |X_c| = \sqrt{L/C}$ $\quad f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad \underline{Z}_{res} = R.$

Close to resonance ($|\Delta f| < 0.1\,f_{res}$) is $\underline{Z} \approx R + j\,X_{res} \cdot 2\,\Delta f / f_{res}$ with $\Delta f = f - f_{res}$

[3] With small loss angle δ (= $1/\varphi$) $\approx$ tan $\delta = -1/(\omega C R)$:

$\underline{Z} = \dfrac{\delta + j}{\omega C}$ $\quad B_{res} = \sqrt{C/L}$: $f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad \underline{Y}_{res} = G.$

[4] Close to resonance ($|\Delta f| < 0.1\,f_{res}$):

$\underline{Y} = G + j\,B_{res} \cdot 2\,\Delta f$ with $\Delta f = f - f_{res}$

[5] e. g. coil with winding capacitance.

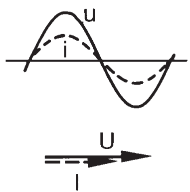
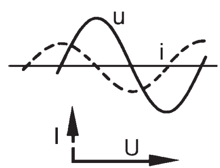
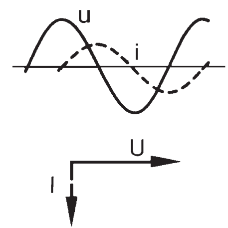
Table 2-6

Current / voltage relationships

		Ohmic resistance R	Capacitance (capacitor) C	Inductance (choke coil) L
General law	$u =$	$i\,R$	$\frac{1}{C}\int i\,\mathrm{d}t$	$L \cdot \frac{\mathrm{d}i}{\mathrm{d}t}$
	$i =$	$\frac{u}{R}$	$C \cdot \frac{\mathrm{d}u}{\mathrm{d}t}$	$\frac{1}{L}\int u\,\mathrm{d}t$
Time law	$u =$	$\hat{u}\sin\omega t$	$\hat{u}\sin\omega t$	$\hat{u}\sin\omega t$
hence	$u =$	$\hat{\imath}\,R\sin\omega t = \hat{u}\sin\omega t$	$-\frac{1}{\omega C}\,\hat{\imath}\cos\omega t = -\hat{u}\cos\omega t$	$\omega L\,\hat{\imath}\cos\omega t = \hat{u}\cos\omega t$
	$i =$	$\frac{\hat{u}}{R}\sin\omega t = \hat{\imath}\sin\omega t$	$\omega C\,\hat{u}\cos\omega t = \hat{\imath}\cos\omega t$	$-\frac{1}{\omega L}\,\hat{u}\cos\omega t = -\hat{\imath}\cos\omega t$
Elements of calculation	$\hat{\imath} =$	$\hat{u}/R$	$\omega C\,\hat{u}$	$\hat{u}/(\omega L)$
	$\hat{u} =$	$\hat{\imath}\,R$	$\hat{\imath}/(\omega C)$	$\hat{\imath}\,\omega L$
	$\varphi =$	0 u and i in phase	$\arctan\frac{1}{\omega C \cdot 0} = -\frac{\pi}{2}$ i leads u by 90 °	$\arctan\frac{\omega L}{0} = \frac{\pi}{2}$ i lags u by 90 °
	$f =$	$\frac{\omega}{2\pi}$	$\frac{\omega}{2\pi}$	$\frac{\omega}{2\pi}$

(continued)

Table 2-6 (continued)

		Ohmic resistance R	Capacitance (capacitor) C	Inductance (choke coil) L
Alternating current impedance	$\underline{Z} =$	R	$\frac{-\,j}{\omega\, C}$	$j\,\omega\, L$
	$\lvert \underline{Z} \rvert =$	R	$\frac{1}{\omega\, C}$	$\omega\, L$
Diagrams				

2.3 Electrical resistances

2.3.1 Definitions and specific values

An ohmic resistance is present if the instantaneous values of the voltage are proportional to the instantaneous values of the current, even in the event of time-dependent variation of the voltage or current. Any conductor exhibiting this proportionality within a defined range (e. g. of temperature, frequency or current) behaves within this range as an ohmic resistance. Active power is converted in an ohmic resistance. For a resistance of this kind is

$$R = \frac{P}{I^2}.$$

The resistance measured with direct current is termed the *d. c. resistance* R_{-}. If the resistance of a conductor differs from the d. c. resistance only as a result of skin effect, we then speak of the a. c. resistance $R_{\sim}$ of the conductor. The ratio expressing the increase in resistance is

$$\zeta = \frac{R_{\sim}}{R_{-}} = \frac{\textit{a. c.}\ \text{resistance}}{\text{d. c. resistance}}.$$

Specific values for major materials are shown in Table 2-7.

Table 2-7

Numerical values for major materials

Conductor	Specific electric resistance ρ ($mm^2\ \Omega/m$)	Electric conductivity $x = 1/\rho$ ($m/mm^2\ \Omega$)	Temperature coefficient α (K^{-1})	Density (kg/dm^3)
Aluminium, 99.5 % Al, soft	0.0278	36	$4 \cdot 20^{-3}$	2.7
Al-Mg-Si	0.03…0.033	33…30	$3.6 \cdot 10^{-3}$	2.7
Al-Mg	0.06…0.07	17…14	$2.0 \cdot 10^{-3}$	2.7
Al bronze, 90 % Cu, 10 % Al	0.13	7.7	$3.2 \cdot 10^{-3}$	8.5
Bismuth	1.2	0.83	$4.5 \cdot 10^{-3}$	9.8
Brass	0.07	14.3	$1.3…1.9 \cdot 10^{-3}$	8.5
Bronze, 88 % Cu, 12 % Sn	0.18	5.56	$0.5 \cdot 10^{-3}$	8.6…9
Cast iron	0.60…1.60	1.67…0.625	$1.9 \cdot 10^{-3}$	7.86…7.2
Conductor copper, soft	0.01754	57	$4.0 \cdot 10^{-3}$	8.92
Conductor copper, hard	0.01786	56	$3.92 \cdot 10^{-3}$	8.92
Constantan	0.49…0.51	2.04…1.96	$-0.05 \cdot 10^{-3}$	8.8
CrAl 20 5	1.37	0.73	$0.05 \cdot 10^{-3}$	—
CrAl 30 5	1.44	0.69	$0.01 \cdot 10^{-3}$	—
Dynamo sheet	0.13	7.7	$4.5 \cdot 10^{-3}$	7.8
Dynamo sheet alloy (1 to 5 % Si)	0.27…0.67	3.7…1.5	—	7.8
Graphite and retort carbon	13…100	0.077…0.01	$-0.8…-0.2 \cdot 10^{-3}$	2.5…1.5
Lead	0.208	4.8	$4.0 \cdot 10^{-3}$	11.35
Magnesium	0.046	21.6	$3.8 \cdot 10^{-3}$	1.74
Manganin	0.43	2.33	$0.01 \cdot 10^{-3}$	8.4
Mercury	0.958	1.04	$0.90 \cdot 10^{-3}$	13.55
Molybdenum	0.054	18.5	$4.3 \cdot 10^{-3}$	10.2
Monel metal	0.42	2.8	$0.19 \cdot 10^{-3}$	—
Nickel silver	0.33	3.03	$0.4 \cdot 10^{-3}$	8.5

(continued)

Table 2-7 (continued)

Numerical values for major materials

Conductor	Specific electric resistance ρ ($mm^2\ \Omega/m$)	Electric conductivity $x = 1/\rho$ ($m/mm^2\ \Omega$)	Temperature coefficient α (K^{-1})	Density (kg/dm^3)
Ni Cr 30 20	1.04	0.96	$0.24 \cdot 10^{-3}$	8.3
Ni Cr 6015	1.11	0.90	$0.13 \cdot 10^{-3}$	8.3
Ni Cr 80 20	1.09	0.92	$0.04 \cdot 10^{-3}$	8.3
Nickel	0.09	11.1	$6.0 \cdot 10^{-3}$	8.9
Nickeline	0.4	2.5	$0.18 \ldots 0.21 \cdot 10^{-3}$	8.3
Platinum	0.1	10	$3.8 \ldots 3.9 \cdot 10^{-3}$	21.45
Red brass	0.05	20	—	8.65
Silver	0.0165	60.5	$41 \cdot 10^{-3}$	10.5
Steel, 0.1% C, 0.5 % Mn	0.13…0.15	7.7…6.7	$4 \ldots 5 \cdot 10^{-3}$	7.86
Steel, 0.25 % C, 0.3 % Si	0.18	5.5	$4 \ldots 5 \cdot 10^{-3}$	7.86
Steel, spring, 0.8 % C	0.20	5	$4 \ldots 5 \cdot 10^{-3}$	7.86
Tantalum	0.16	6.25	$3.5 \ldots 10^{-3}$	16.6
Tin	0.12	8.33	$4.4 \cdot 10^{-3}$	7.14
Tungsten	0.055	18.2	$4.6 \cdot 10^{-3}$	19.3
Zinc	0.063	15.9	$3.7 \cdot 10^{-3}$	7.23

Resistance varies with temperature, cf. Section 2.3.3

2.3.2 Resistances in different circuit configurations

Connected in series (Fig. 2-4)

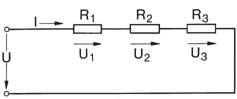


Fig. 2-4

Total resistance = Sum of individual resistances

$$R = R_1 + R_2 + R_3 + \ldots$$

The component voltages behave in accordance with the resistances $U_1 = I\,R_1$ etc.

The current at all resistances is of equal magnitude $I = \dfrac{U}{R}$.

Connected in parallel (Fig. 2-5)

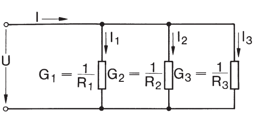


Fig. 2-5

Total conductance = Sum of the individual conductances

$$\frac{1}{R} = G = G_1 + G_2 + G_3 + \ldots \qquad R = \frac{1}{G}.$$

In the case of n equal resistances the total resistance is the nth part of the individual resistances. The voltage at all the resistances is the same. Total current

$$I = \frac{U}{R} = \text{Sum of components } I_1 = \frac{U}{R_1} \text{ etc.}$$

The currents behave inversely to the resistances

$$I_1 = I\frac{R}{R_1};\ I_2 = I\frac{R}{R_2};\ I_3 = I\frac{R}{R_3}.$$

Transformation delta-star and star-delta (Fig. 2-6)

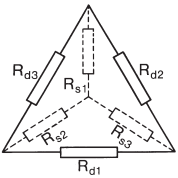


Fig. 2-6

Conversion from delta to star connection with the same total resistance:

$$R_{S1} = \frac{R_{d2}\,R_{d3}}{R_{d1} + R_{d2} + R_{d3}}$$

$$R_{S2} = \frac{R_{d3}\,R_{d1}}{R_{d1} + R_{d2} + R_{d3}}$$

$$R_{S3} = \frac{R_{d1}\,R_{d2}}{R_{d1} + R_{d2} + R_{d3}}$$

Conversion from star to delta connection with the same total resistance:

$$R_{d1} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S1}}$$

$$R_{d2} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S2}}$$

$$R_{d3} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S3}}$$

Calculation of a bridge between points A and B (Fig. 2-7)

To be found:

1. the total resistance R_{tot} between points A and B,
2. the total current I_{tot} between points A and B,
3. the component currents in R_1 to R_5.

Given:

voltage $U = 220$ V.
resistance $R_1 = 10\ \Omega$,
$R_2 = 20\ \Omega$,
$R_3 = 30\ \Omega$,
$R_4 = 40\ \Omega$,
$R_5 = 50\ \Omega$.

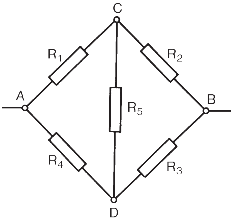


Fig. 2-7

First delta connection CDB is converted to star connection CSDB (Fig. 2-8):

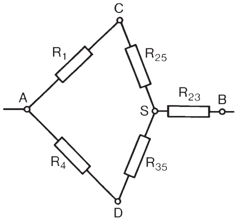


Fig. 2-8

$$R_{25} = \frac{R_2\,R_5}{R_2 + R_3 + R_5} = \frac{20 \cdot 50}{20 + 30 + 50} = 10\ \Omega,$$

$$R_{35} = \frac{R_3\,R_5}{R_2 + R_3 + R_5} = \frac{30 \cdot 50}{20 + 30 + 50} = 15\ \Omega,$$

$$R_{23} = \frac{R_2\,R_3}{R_2 + R_3 + R_5} = \frac{20 \cdot 30}{20 + 30 + 50} = 6\ \Omega,$$

$$R_{tot} = \frac{(R_1 + R_{25})\,(R_4 + R_{35})}{R_1 + R_{25} + R_4 + R_{35}} + R_{23} =$$

$$= \frac{(10 + 10)\,(40 + 15)}{10 + 10 + 40 + 15} + 6 = 20.67\ \Omega.$$

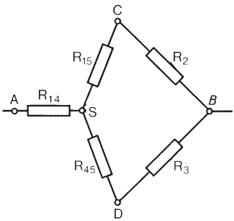


Fig. 2-9

$$I_{tot} = \frac{U}{R_{tot}} = \frac{220}{20.67} = 10.65\ \text{A}.$$

$$I_{R1} = I_{tot}\,\frac{R_{tot} - R_{23}}{R_1 + R_{25}} = 10.65 \cdot \frac{20.67 - 6}{10 + 10} = 7.82\ \text{A},$$

$$I_{R4} = I_{tot}\,\frac{R_{tot} - R_{23}}{R_4 + R_{35}} = 10.65 \cdot \frac{20.67 - 6}{40 + 15} = 2.83\ \text{A},$$

By converting the delta connection CDA to star connection CSDA, we obtain the following values (Fig. 2-9): $R_{15} = 5\,\Omega$; $R_{45} = 20\,\Omega$; $R_{14} = 4\,\Omega$; $I_{R2} = 7.1$ A; $I_{R3} = 3.55$ A.

With alternating current the calculations are somewhat more complicated and are carried out with the aid of resistance operators. Using the symbolic method of calculation, however, it is basically the same as above.

2.3.3 The influence of temperature on resistance

The resistance of a conductor is

$$R = \frac{l \cdot \rho}{A} = \frac{l}{x \cdot A}$$

where

l = Total length of conductor

A = Cross-sectional area of conductor

ρ = Specific resistance (at 20 °C)

$x = \frac{1}{\rho}$ Conductance

α = Temperature coefficient.

Values for ρ, x and α are given in Table 2-7 for a temperature of 20 °C.

For other temperatures ϑ[1] (ϑ in °C)

$$\rho_\vartheta = \rho_{20}\,[1 + \alpha\,(\vartheta - 20)]$$

[1] Valid for temperatures from – 50 to + 200 °C.

and hence for the conductor resistance

$$R_\vartheta = \frac{l}{A} \cdot \rho_{20} [1 + \alpha (\vartheta - 20)].$$

Similarly for the conductivity

$$x_\vartheta = x_{20} [1 + \alpha (\vartheta - 20)]^{-1}$$

The temperature rise of a conductor or a resistance is calculated as

$$\Delta \vartheta = \frac{R_w / R_k - 1}{\alpha}.$$

The values R_k and R_w are found by measuring the resistance of the conductor or resistance in the cold and hot conditions, respectively.

Example:

The resistance of a copper conductor of $l = 100$ m and $A = 10$ mm² at 20 °C is

$$R_{20} = \frac{100 \cdot 0.0175}{10} = 0.175\ \Omega.$$

If the temperature of the conductor rises to $\vartheta = 50$ °C, the resistance becomes

$$R_{50} = \frac{100}{10} \cdot 0.0175 [1 + 0.004 (50 - 20)] \approx 0.196\ \Omega.$$

2.4 Relationships between voltage drop, power loss and conductor cross section

Especially in low-voltage networks is it necessary to check that the conductor cross-section, chosen with respect to the current-carrying capacity, is adequate as regards the voltage drop. It is also advisable to carry out this check in the case of very long connections in medium-voltage networks. (See also Sections 6.1.6 and 13.2.3).

Direct current

voltage drop $$\Delta U = R'_L \cdot 2 \cdot l \cdot I = \frac{2 \cdot l \cdot I}{x \cdot A} = \frac{2 \cdot l \cdot P}{x \cdot A \cdot U}$$

percentage voltage drop $$\Delta u = \frac{\Delta U}{U_n}\ 100\ \% = \frac{R'_L \cdot 2 \cdot l \cdot I}{U_n}\ 100\ \%$$

power loss $$\Delta P = I^2 R'_L\, 2 \cdot l = \frac{2 \cdot l \cdot P^2}{x \cdot A \cdot U^2}$$

percentage power loss $$\Delta p = \frac{\Delta P}{P_n}\ 100\ \% = \frac{I^2 R'_L \cdot 2 \cdot l}{P_n}\ 100\ \%$$

conductor cross section $$A = \frac{2 \cdot l \cdot I}{x \cdot \Delta U} = \frac{2 \cdot l \cdot I}{x \cdot \Delta u \cdot U}\ 100\ \% = \frac{2 \cdot l \cdot P}{\Delta p \cdot U^2 \cdot x}\ 100\ \%$$

Single-phase alternating current

voltage drop[2)]
$$\Delta U = I \cdot 2 \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)$$

percentage voltage drop[2)]
$$\Delta u = \frac{\Delta U}{U_n} 100\,\% = \frac{I \cdot 2 \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)}{U_n}$$

power loss
$$\Delta P = I^2 R'_L \cdot 2 \cdot l = \frac{2 \cdot l \cdot P^2}{x \cdot A \cdot U^2 \cdot \cos^2\varphi}$$

percentage power loss
$$\Delta p = \frac{\Delta P}{P_n} 100\,\% = \frac{I^2 \cdot R'_L \cdot 2 \cdot l}{P_n} 100\,\%$$

conductor cross-section[1)]
$$A = \frac{2 \cdot l \cos\varphi}{x\left(\frac{\Delta U}{I} - X'_L \cdot 2 \cdot l \cdot \sin\varphi\right)} = \frac{2 \cdot l \cos\varphi}{x\left(\frac{\Delta u \cdot U_n}{I \cdot 100\,\%} - X'_L \cdot 2 \cdot l \cdot \sin\varphi\right)}$$

Three-phase current

voltage drop[2)]
$$\Delta U = \sqrt{3} \cdot I \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)$$

percentage voltage drop[2)]
$$\Delta u = \frac{\Delta U}{U_n} 100\,\% = \frac{\sqrt{3} \cdot I \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)}{U_n} 100\,\%$$

power loss
$$\Delta P = 3 \cdot I^2 R'_L \cdot l = \frac{l \cdot P^2}{x \cdot A \cdot U^2 \cdot \cos^2\varphi}$$

percentage power loss
$$\Delta p = \frac{\Delta P}{P_n} 100\,\% = \frac{3\,I^2 \cdot R'_L \cdot l}{P_n} 100\,\%$$

conductor cross-section[1)]
$$A = \frac{l \cdot \cos\varphi}{x\left(\frac{\Delta U}{\sqrt{3} \cdot I} - X'_L \cdot l \cdot \sin\varphi\right)} = \frac{l \cdot \cos\varphi}{x\left(\frac{\Delta u \cdot U}{\sqrt{3} \cdot I \cdot 100\,\%} - X'_L \cdot l \cdot \sin\varphi\right)}$$

l = one-way length of conductor

R'_L = Resistance per km

P = Active power to be transmitted ($P = P_n$)

U = phase-to-phase voltage

X'_L = Reactance per km

I = phase-to-phase current

In single-phase and three-phase a.c. systems with cables and lines of less than 16 mm² the inductive reactance can usually be disregarded. It is sufficient in such cases to calculate only with the d.c. resistance.

1) Reactance is slightly dependent on conductor cross section.

2) Longitudinal voltage drop becomes effectively apparent.

Table 2-8

Effective resistances per unit length of PVC-insulated cables with copper conductors as per DIN VDE 0271 for 0.6/1 kV

Number of conductors and cross-section	D. C. resistance at 70 °C	Ohmic resistance at 70 °C	Inductive reactance	Effective resistance per unit length $R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi$ at $\cos\varphi$ 0.95	0.9	0.8	0.7	0.6
	R'_{L-}	$R'_{L\sim}$	X'_L					
mm²	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km
4 × 1.5	14.47	14.47	0.115	13.8	13.1	11.65	10.2	8.77
4 × 2.5	8.71	8.71	0.110	8.31	7.89	7.03	6.18	5.31
4 × 4	5.45	5.45	0.107	5.21	4.95	4.42	3.89	3.36
4 × 6	3.62	3.62	0.100	3.47	3.30	2.96	2.61	2.25
4 × 10	2.16	2.16	0.094	2.08	1.99	1.78	1.58	1.37
4 × 16	1.36	1.36	0.090	1.32	1.26	1.14	1.020	0.888
4 × 25	0.863	0.863	0.086	0.847	0.814	0.742	0.666	0.587
4 × 35	0.627	0.627	0.083	0.622	0.60	0.55	0.498	0.443
4 × 50	0.463	0.463	0.083	0.466	0.453	0.42	0.38	0.344
4 × 70	0.321	0.321	0.082	0.331	0.326	0.306	0.283	0.258
4 × 95	0.231	0.232	0.082	0.246	0.245	0.235	0.221	0.205
4 × 120	0.183	0.184	0.080	0.2	0.2	0.195	0.186	0.174
4 × 150	0.149	0.150	0.080	0.168	0.17	0.168	0.162	0.154
4 × 185	0.118	0.1202	0.080	0.139	0.143	0.144	0.141	0.136
4 × 240	0.0901	0.0922	0.079	0.112	0.117	0.121	0.121	0.119
4 × 300	0.0718	0.0745	0.079	0.0954	0.101	0.107	0.109	0.108

Example:

A three-phase power of 50 kW with $\cos\varphi = 0.8$ is to be transmitted at 400 V over a line 100 m long. The voltage drop must not exceed 2 %. What is the required cross section of the line?

The percentage voltage drop of 2 % is equivalent to

$$\Delta U = \frac{\Delta u}{100\,\%} U_n = \frac{2\,\%}{100\,\%} 400\text{ V} = 8.0\text{ V}.$$

The current is

$$I = \frac{P}{\sqrt{3} \cdot U \cdot \cos\varphi} = \frac{50\text{ kW}}{\sqrt{3} \cdot 400\text{ V} \cdot 0.8} = 90\text{ A}.$$

Calculation is made easier by Table 2-8, which lists the effective resistance per unit length $R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi$ for the most common cables and conductors. Rearranging the formula for the voltage drop yields

$$R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi = \frac{\Delta U}{\sqrt{3} \cdot I \cdot l} = \frac{8.0}{\sqrt{3} \cdot 90\text{ A} \cdot 0.1\text{ km}} = 0.513\ \Omega/\text{km}.$$

According to Table 2-8 a cable of 50 mm² with an effective resistance per unit length of 0.42 Ω/km should be used. The actual voltage drop will then be

$$\Delta U = \sqrt{3} \cdot I \cdot l \,(R_L' \cdot \cos\varphi + X_L' \cdot \sin\varphi)$$
$$= \sqrt{3} \cdot 90\ \text{A} \cdot 0.1\ \text{km} \cdot 0.42\ \Omega/\text{km} = 6.55\ \text{V}.$$

This is equivalent to $\Delta u = \frac{\Delta U}{U_n} 100\ \% = \frac{6.55\ \text{V}}{400\ \text{V}} 100\ \% = 1.6\ \%.$

2.5 Current input of electrical machines and transformers

Direct current

Motors:

$$I = \frac{P_{mech}}{U \cdot \eta}$$

Generators:

$$I = \frac{P}{U}$$

Single-phase alternating current

Motors:

$$I = \frac{P_{mech}}{U \cdot \eta \cdot \cos\varphi}$$

Transformers and synchronous generators:

$$I = \frac{S}{U}$$

Three-phase current

Induction motors:

$$I = \frac{P_{mech}}{\sqrt{3} \cdot U \cdot \eta \cdot \cos\varphi}$$

Transformers and synchronous generators:

$$I = \frac{S}{\sqrt{3} \cdot U}$$

Synchronous motors:

$$I \approx \frac{P_{mech}}{\sqrt{3} \cdot U \cdot \eta \cdot \cos\varphi} \cdot \sqrt{1 + \tan^2\varphi}$$

In the formulae for three-phase current, U is the phase voltage.

Table 2-9

Motor current ratings for three-phase motors (typical values for squirrel-cage type)

Smallest possible short-circuit fuse (Service category gG[1)]) for three-phase motors. The maximum value is governed by the switching device or motor relay.

Motor output data			Rated currents at							
			230 V		400 V		500 V		600 V	
			Motor	Fuse	Motor	Fuse	Motor	Fuse	Motor	Fuse
kW	cos φ	η %	A	A	A	A	A	A	A	A
0.25	0.7	62	1.4	4	0.8	2	0.6	2	—	—
0.37	0.72	64	2.0	4	1.2	4	0.9	2	0.7	2
0.55	0.75	69	2.7	4	1.5	4	1.2	4	0.9	2
0.75	0.8	74	3.2	6	1.8	4	1.5	4	1.1	2
1.1	0.83	77	4.3	6	2.5	4	2	4	1.5	2
1.5	0.83	78	5.8	16	3.3	6	2.6	4	2	4
2.2	0.83	81	8.2	20	4.7	10	3.7	10	2.9	6
3	0.84	81	11.1	20	6.4	16	5	10	3.5	6

(continued)

Table 2-9 (continued)

Motor current ratings for three-phase motors (typical values for squirrel-cage type)

Smallest possible short-circuit fuse (Service category gG[1]) for three-phase motors. The maximum value is governed by the switching device or motor relay.

Motor output data			Rated currents at							
			230 V		400 V		500 V		660 V	
			Motor	Fuse	Motor	Fuse	Motor	Fuse	Motor	Fuse
kW	cos φ	η %	A	A	A	A	A	A	A	A
4	0.84	82	14.6	25	8.4	20	6.4	16	4.9	10
5.5	0.85	83	19.6	35	11.3	25	8.6	20	6.7	16
7.5	0.86	85	25.8	50	14.8	35	11.5	25	9	16
11	0.86	87	36.9	63	21.2	35	17	35	13	25
15	0.86	87	50	80	29	50	22.5	35	17.5	25
18.5	0.86	88	61	100	35	63	27	50	21	35
22	0.87	89	71	100	41	63	32	63	25	35
30	0.87	90	96	125	55	80	43	63	33	50
37	0.87	90	119	200	68	100	54	80	42	63
45	0.88	91	141	225	81	125	64	100	49	63
55	0.88	91	172	250	99	160	78	125	60	100
75	0.88	91	235	350	135	200	106	160	82	125
90	0.88	92	279	355	160	225	127	200	98	125
110	0.88	92	341	425	196	250	154	225	118	160
132	0.88	92	409	600	235	300	182	250	140	200
160	0.88	93	491	600	282	355	220	300	170	224
200	0.88	93	613	800	353	425	283	355	214	300
250	0.88	93	—	—	441	500	355	425	270	355
315	0.88	93	—	—	556	630	444	500	337	400
400	0.89	96	—	—	—	—	534	630	410	500
500	0.89	96	—	—	—	—	—	—	515	630

[1] see 7.1.2 for definitions

The motor current ratings relate to normal internally cooled and surface-cooled threephase motors with synchronous speeds of 1500 min^{-1}.

The fuses relate to the stated motor current ratings and to direct starting:
starting current max. 6 × rated motor current,
starting time max. 5 s.

In the case of slipring motors and also squirrel-cage motors with star-delta starting ($t_{start} \leqq 15$ s, $I_{start} = 2 \cdot I_n$) it is sufficient to size the fuses for the rated current of the motor concerned.

Motor relay in phase current: set to 0.58 × motor rated current.

With higher rated current, starting current and/or longer starting time, use larger fuses. Note comments on protection of lines and cables against overcurrents (Section 13.2.3).

2.6 Attenuation constant *a* of transmission systems

The transmission properties of transmission systems, e. g. of lines and two-terminal pair networks, are denoted in logarithmic terms for the ratio of the output quantity to the input quantity of the same dimension. When several transmission elements are arranged in series the total attenuation or gain is then obtained, again in logarithmic terms, by simply adding together the individual partial quantities.

The natural logarithm for the ratio of two quantities, e. g. two voltages, yields the voltage gain in Neper (Np):

$$\frac{a}{\mathrm{Np}} = \ln U_2/U_1.$$

If $P = U^2/R$, the power gain, provided $R_1 = R_2$ is

$$\frac{a}{\mathrm{Np}} = \frac{1}{2} \ln P_2/P_1.$$

The conversion between logarithmic ratios of voltage, current and power when $R_1 \neq R_2$ is

$$\ln U_2/U_1 = \ln I_2/I_1 + \ln R_2/R_1 = \frac{1}{2} \ln P_2/P_1 + \frac{1}{2} \ln R_2/R_1.$$

The common logarithm of the power ratio is the power gain in Bel. It is customary to calculate with the decibel (dB), one tenth of a Bel:

$$\frac{a}{\mathrm{dB}} = 10 \lg P_2/P_1.$$

If $R_1 = R_2$, for the conversion we have

$$\frac{a}{\mathrm{dB}} = 20 \lg U_2/U_1 \text{ respectively } \frac{a}{\mathrm{dB}} = 20 \lg I_2/I_1.$$

If $R_1 \neq R_2$, then

$$10 \lg P_2/P_1 = 20 \lg U_2/U_1, - 10 \lg R_2/R_1, = 20 \lg I_2/I_1, + 10 \lg R_2/R_1.$$

Relationship between Neper and decibel:

1 dB = 0.1151 Np
1 Np = 8.6881 dB

In the case of absolute levels one refers to the internationally specified values P_0 = 1 mW at 600 Ω, equivalent to $U_0 \cdot 0.775$ V, $I_0 \cdot 1.29$ mA (0 Np or 0 dB).

For example, 0.36 Np signifies a voltage ratio of $U/U_0 = e^{0.35} = 1.42$.

This corresponds to an absolute voltage level of $U = 0.776 \text{ V} \cdot 1.42 = 1.1$ V. Also 0.35 Np = 0.35 · 8.6881 = 3.04 dB.

3 Calculation of Short-Circuit Currents in Three-Phase Systems

3.1 Terms and definitions

3

3.1.1 Terms as per DIN VDE 0102 / IEC 909

Short circuit: the accidental or deliberate connection across a comparatively low resistance or impedance between two or more points of a circuit which usually have differing voltage.

Short-circuit current: the current in an electrical circuit in which a short circuit occurs.

Prospective (available) short-circuit current: the short-circuit current which would arise if the short circuit were replaced by an ideal connection having negligible impedance without alteration of the incoming supply.

Symmetrical short-circuit current: root-mean-square (r.m.s.) value of the symmetrical alternating-current (a.c.) component of a prospective short-circuit current, taking no account of the direct-current (d.c.) component, if any.

Initial symmetrical short-circuit current I_k''*:* the r.m.s. value of the symmetrical a.c. component of a prospective short-circuit current at the instant the short circuit occurs if the short-circuit impedance retains its value at time zero.

Initial symmetrical (apparent) short-circuit power S_k''*:* a fictitious quantity calculated as the product of initial symmetrical short-circuit current I_k'', nominal system voltage U_n and the factor $\sqrt{3}$.

D.C. (aperiodic) component i_{DC} *of short-circuit current:* the mean value between the upper and lower envelope curve of a short-circuit current decaying from an initial value to zero.

Peak short-circuit current i_p*:* the maximum possible instantaneous value of a prospective short-circuit current.

Symmetrical short-circuit breaking current I_a*:* the r.m.s. value of the symmetrical a.c. component of a prospective short-circuit current at the instant of contact separation by the first phase to clear of a switching device.

Steady-state short-circuit current I_k*:* the r.m.s. value of the symmetrical a.c. component of a prospective short-circuit current persisting after all transient phenomena have died away.

(Independent) Voltage source: an active element which can be simulated by an ideal voltage source in series with a passive element independently of currents and other voltages in the network.

Nominal system voltage U_n*:* the (line-to-line) voltage by which a system is specified and to which certain operating characteristics are referred.

Equivalent voltage source $cU_n / \sqrt{3}$*:* the voltage of an ideal source applied at the short-circuit location in the positive-sequence system as the network's only effective voltage in order to calculate the short-circuit currents by the equivalent voltage source method.

Voltage factor c: the relationship between the voltage of the equivalent voltage source and $U_n / \sqrt{3}$.

Subtransient voltage E" of a synchronous machine: the r.m.s. value of the symmetrical interior voltages of a synchronous machine which is effective behind the subtransient reactance X_d'' at the instant the short circuit occurs.

Far-from-generator short circuit: a short circuit whereupon the magnitude of the symmetrical component of the prospective short-circuit current remains essentially constant.

Near-to-generator short circuit: a short circuit whereupon at least one synchronous machine delivers an initial symmetrical short-circuit current greater than twice the synchronous machine's rated current, or a short circuit where synchronous or induction motors contribute more than 5 % of the initial symmetrical short-circuit current I_k'' without motors.

Positive-sequence short-circuit impedance $\underline{Z}_{(1)}$ *of a three-phase a.c. system:* the impedance in the positive-phase-sequence system as viewed from the fault location.

Negative-sequence short-circuit impedance $\underline{Z}_{(2)}$ *of a three-phase a.c. system:* the impedance in the negative-phase-sequence system as viewed from the fault location.

Zero-sequence short-circuit impedance $\underline{Z}_{(0)}$ *of a three-phase a.c. system:* the impedance in the zero-phase-sequence system as viewed from the fault location. It includes the threefold value of the neutral-to-earth impedance.

Subtransient reactance X_d'' *of a synchronous machine:* the reactance effective at the instant of the short circuit. For calculating short-circuit currents, use the saturated value X_d''.

Minimum time delay t_{min} *of a circuit-breaker:* the shortest possible time from commencement of the short-circuit current until the first contacts separate in one pole of a switching device.

3.1.2 Symmetrical components of asymmetrical three-phase systems

In three-phase networks a distinction is made between the following kinds of fault:

a) three-phase fault (I''_{k3})
b) phase-to-phase fault clear of ground (I''_{k2})
c) two-phase-to-earth fault (I''_{k2E}; I''_{kE2E})
d) phase-to-earth fault (I''_{k1})
e) double earth fault (I''_{kEE})

A 3-phase fault affects the three-phase network symmetrically. All three conductors are equally involved and carry the same rms short-circuit current. Calculation need therefore be for only one conductor.

All other short-circuit conditions, on the other hand, incur asymmetrical loadings. A suitable method for investigating such events is to split the asymmetrical system into its symmetrical components.

With a symmetrical voltage system the currents produced by an asymmetrical loading (I_1, I_2 and I_3) can be determined with the aid of the symmetrical components (positive-, negative- and zero-sequence system).

The symmetrical components can be found with the aid of complex calculation or by graphical means.

We have:

Current in pos.-sequence system $\underline{I}_m = \frac{1}{3}(\underline{I}_1 + \underline{a}\ I_2 + \underline{a}^2 \underline{I}_3)$

Current in neg.-sequence system $\underline{I}_g = \frac{1}{3}(\underline{I}_1 + \underline{a}^2 \underline{I}_2 + \underline{a}\ \underline{I}_3)$

Current in zero-sequence system $\underline{I}_o = \frac{1}{3}(\underline{I}_1 + \underline{I}_2 + \underline{I}_3)$

For the rotational operators of value 1:

$$\underline{a} = e^{j120^\circ};\ \underline{a}^2 = e^{j240^\circ};\ 1 + \underline{a} + \underline{a}^2 = 0$$

The above formulae for the symmetrical components also provide information for a graphical solution.

If the current vector leading the current in the reference conductor is rotated 120° *backwards,* and the lagging current vector 120 ° *forwards,* the resultant is equal to three times the vector $\underline{I}_m$ in the reference conductor. The negative-sequence components are apparent.

If one turns in the other direction, the positive-sequence system is evident and the resultant is three times the vector $\underline{I}_g$ in the reference conductor.

Geometrical addition of all three current vectors ($\underline{I}_1$, $\underline{I}_2$ and $\underline{I}_3$) yields three times the vector $\underline{I}_0$ in the reference conductor.

If the neutral conductor is unaffected, there is no zero-sequence system.

3.2 Fundamentals of calculation according to DIN VDE 0102 / IEC 909

In order to select and determine the characteristics of equipment for electrical networks it is necessary to know the magnitudes of the short-circuit currents and short-circuit powers which may occur.

The short-circuit current at first runs asymmetrically to the zero line, Fig. 3-1. It contains an alternating-current component and a direct-current component.

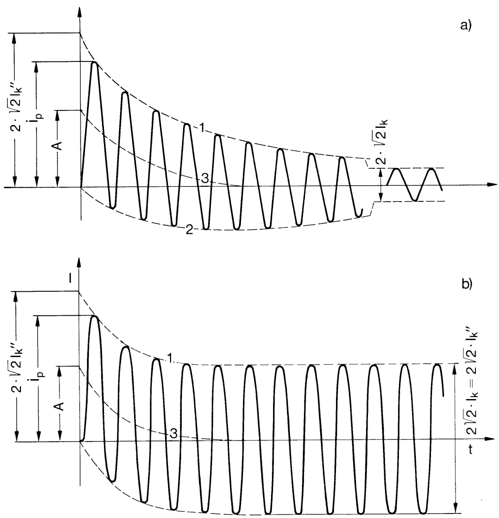


Fig. 3-1

Curve of short-circuit current: a) near-to-generator fault, b) far-from-generator fault I_k'' initial symmetrical short-circuit current, i_p peak short-circuit current, I_k steady state short-circuit current, A initial value of direct current, 1 upper envelope, 2 lower envelope, 3 decaying direct current.

Calculatlon of initial symmetrical short-circuit current I_k''

The calculation of short-circuit currents is always based on the assumption of a dead short circuit. Other influences, especially arc resistances, contact resistances, conductor temperatures, inductances of current transformers and the like, can have the effect of lowering the short-circuit currents. Since they are not amenable to calculation, they are accounted for in Table 3-1 by the factor c.

Initial symmetrical short-circuit currents are calculated with the equations in Table 3-2.

Table 3-1

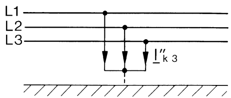
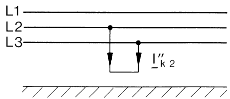
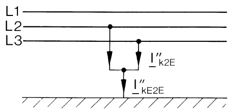
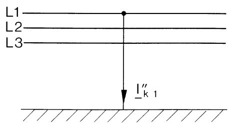
Voltage factor *c*

Nominal voltage	Voltage factor c for calculating the greatest short-circuit current c_{max}	 the smallest short-circuit current c_{min}
Low voltage		
100 V to 1000 V		
(see IEC 38, Table I)		
a) 230 V / 400 V	1.00	0.95
b) other voltages	1.05	1.00
Medium voltage		
>1 kV to 35 kV	1.10	1.00
(see IEC 38, Table III)		
High-voltage		
> 35 kV to 230 kV	1.10	1.00
(see IEC 38, Table IV)		
380 kV	1.10	1.00

Note: cU_n should not exceed the highest voltage U_m for power system equipment.

Table 3-2

Formulae for calculating initial short-circuit current and short-circuit powers

Kind of fault		Dimension equations (IEC 909)	Numerical equations of the % / MVA systems
Three-phase fault with or without earth fault		$I''_{k3} = \dfrac{1.1 \cdot U_n}{\sqrt{3}\,\lvert \underline{Z}_1 \rvert}$ $S''_k = \sqrt{3}\, U_n\, I''_{k3}$	$I''_{k3} = \dfrac{1.1 \cdot 100\,\%}{\lvert \sqrt{3}\, \underline{Z}_1 \rvert} \cdot \dfrac{1}{U_n}$ $S''_k = \dfrac{1.1 \cdot 100\,\%}{\underline{Z}_1}$
Phase-to-phase fault clear of ground		$I''_{k2} = \dfrac{1.1 \cdot U_n}{\lvert \underline{Z}_1 + \underline{Z}_2 \rvert}$	$I''_{k2} = \dfrac{1.1 \cdot 100\,\%}{\lvert \underline{Z}_1 + \underline{Z}_2 \rvert} \cdot \dfrac{1}{U_n}$
Two-phase-to-earth fault		$I''_{kE2E} = \dfrac{\sqrt{3} \cdot 1.1\, U_n}{\left\lvert \underline{Z}_1 + \underline{Z}_0 + \underline{Z}_0 \dfrac{\underline{Z}_1}{\underline{Z}_2} \right\rvert}$	$I''_{kE2E} = \dfrac{\sqrt{3} \cdot 1.1 \cdot 100\,\%}{\left\lvert \underline{Z}_1 + \underline{Z}_0 + \underline{Z}_0 \dfrac{\underline{Z}_1}{\underline{Z}_2} \right\rvert} \cdot \dfrac{1}{U_n}$
Phase-to-earth fault		$I''_{k1} = \dfrac{\sqrt{3} \cdot 1.1 \cdot U_n}{\lvert \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_0 \rvert}$	$I''_{k1} = \dfrac{\sqrt{3} \cdot 1.1 \cdot 100\,\%}{\lvert \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_0 \rvert} \cdot \dfrac{1}{U_n}$

In the right-hand column of the Table, I''_k is in kA, S''_k in MVA, U_n in kV and Z in % / MVA.
The directions of the arrows shown here are chosen arbitrarily.

Calculation of peak short-circuit current i_p

When calculating the peak short-circuit current i_p, sequential faults are disregarded. Three-phase short circuits are treated as though the short circuit occurs in all three conductors simultaneously. We have:

$$i_p = \kappa \cdot \sqrt{2} \cdot I''_k.$$

The factor κ takes into account the decay of the d. c. component. It can be calculated as

$\kappa = 1.02 + 0.98\ e^{-3\ R/X}$ or taken from Fig. 3-2.

Exact calculation of i_p with factor κ is possible only in networks with branches having the same ratios R/X. If a network includes parallel branches with widely different ratios R/X, the following methods of approximation can be applied:

a) Factor κ is determined uniformly for the smallest ratio R/X. One need only consider the branches which are contained in the faulted network and carry partial short-circuit currents.

b) The factor is found for the ratio R/X from the resulting system impedance $Z_k = R_k + jX_k$ at the fault location, using $1.15 \cdot \kappa_k$ for calculating i_p. In low-voltage networks the product $1.15 \cdot \kappa$ is limited to 1.8, and in high-voltage networks to 2.0.

c) Factor κ can also be calculated by the method of the equivalent frequency as in IEC 909 para. 9.1.3.2.

The maximum value of $\kappa = 2$ is attained only in the theoretical limiting case with an active resistance of $R = 0$ in the short-circuit path. Experience shows that with a short-circuit at the generator terminals a value of $\kappa = 1.8$ is not exceeded with machines < 100 MVA.

With a unit-connected generator and high-power transformer, however, a value of $\kappa = 1.9$ can be reached in unfavourable circumstances in the event of a short circuit near the transformer on its high-voltage side, owing to the transformer's very small ratio R/X. The same applies to networks with a high fault power if a short circuit occurs after a reactor.

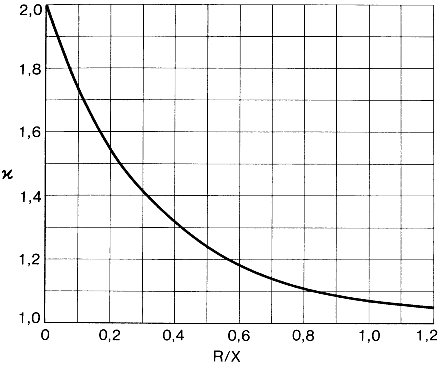


Fig. 3-2

Factor κ

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Calculation of steady-state short-circuit current I_k

Three-phase fault with single supply

$I_k = I''_{kQ}$	network
$I_k = \lambda \cdot I_{rG}$	synchronous machine

Three-phase fault with single supply from more than one side

$I_k = I_{bkW} + I''_{kQ}$

I_{bkw}	symmetrical short-circuit breaking current of a power plant
I''_{kQ}	initial symmetrical short-circuit current of network

Three-phase fault in a meshed network

$I_k = I''_{koM}$

I''_{koM}	initial symmetrical short-circuit current without motors

I_k depends on the excitation of the generators, on saturation effects and on changes in switching conditions in the network during the short circuit. An adequate approximation for the upper and lower limit values can be obtained with the factors λ_{max} and λ_{min}, Fig. 3-3 and 3-4. I_{rG} is the rated current of the synchronous machine.

For X_{dsat} one uses the reciprocal of the no-load/short-circuit ratio I_{k0}/I_{rG}(VDE 0530 Part 1).

The 1st series of curves of λ_{max} applies when the maximum excitation voltage reaches 1.3 times the excitation voltage for rated load operation and rated power factor in the case of turbogenerators, or 1.6 times the excitation for rated load operation in the case of salient-pole machines.

The 2nd series of curves of λ_{max} applies when the maximum excitation voltage reaches 1.6 times the excitation for rated load operation in the case of turbogenerators, or 2.0 times the excitation for rated load operation in the case of salient-pole machines.

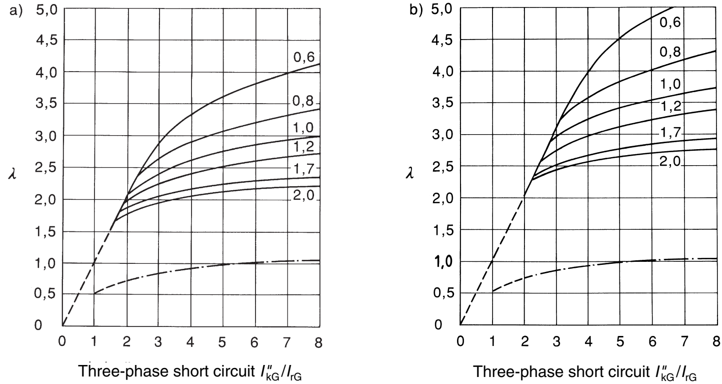


Fig. 3-3

Factors λ for salient-pole machines in relation to ratio I''_{kG}/I_{rG} and saturated synchronous reactance X_d of 0.6 to 2.0, —— λ_{max}, – · – λ_{min};
a) Series 1 U_{fmax}/U_{fr} = 1.6; b) Series 2 U_{fmax}/U_{fr} = 2.0.

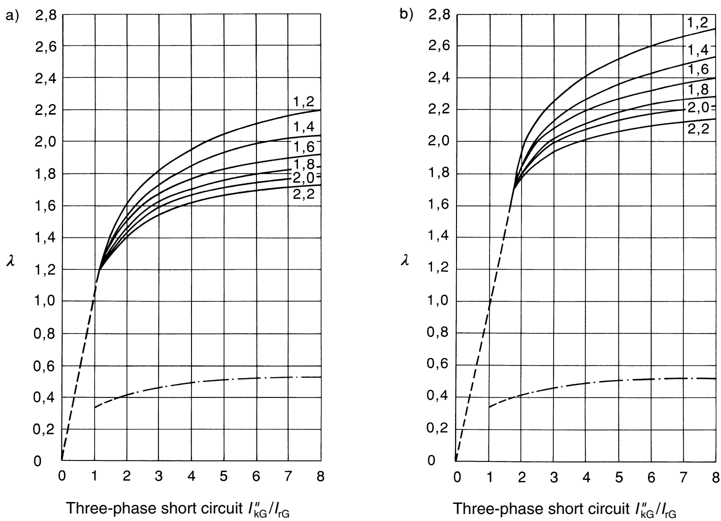


Fig. 3-4

Factors λ for turbogenerators in relation to ratio I''_{kG}/I_{rG} and saturated synchronous reactance X_d of 1.2 to 2.2, —— λ_{max}, – · – λ_{min};
a) Series 1 U_{fmax}/U_{fr} = 1.3; b) Series 2 U_{fmax}/U_{fr} = 1.6.

Calculation of symmetrical breaking current I_a

Three-phase fault with single supply

$I_a = \mu \cdot I''_{kG}$	synchronous machine
$I_a = \mu \cdot q \cdot I''_{kM}$	induction machine
$I_a = I''_{kQ}$	network

Three-phase fault with single supply from more than one side

$$I_a = I_{aKW} + I''_{kQ} + I_{aM}$$

I_{aKW}	symmetrical short-circuit breaking current of a power plant
I_{kQ}	initial symmetrical short-circuit current of a network
I_{aM}	symmetrical short-circuit breaking current of an induction machine

Three-phase fault in a meshed network

$$I_a = I''_k$$

A more exact result for the symmetrical short-circuit breaking current is obtained with IEC 909 section 12.2.4.3, equation (60).

The factor μ denotes the decay of the symmetrical short-circuit current during the switching delay time. It can be taken from Fig. 3-5 or the equations.

$$\mu = 0.84 + 0.26\ e^{-0.26\ I''_{kG}/\ I_{rG}} \text{ for } t_{min} = 0.02 \text{ s}$$

$$\mu = 0.71 + 0.51\ e^{-0.30\ I''_{kG}/\ I_{rG}} \text{ for } t_{min} = 0.05 \text{ s}$$

$$\mu = 0.62 + 0.72\ e^{-0.32\ I''_{kG}/\ I_{rG}} \text{ for } t_{min} = 0.10 \text{ s}$$

$$\mu = 0.56 + 0.94\ e^{-0.38\ I''_{kG}/\ I_{rG}} \text{ for } t_{min} = 0.25 \text{ s}$$

$$\mu_{max} = 1$$

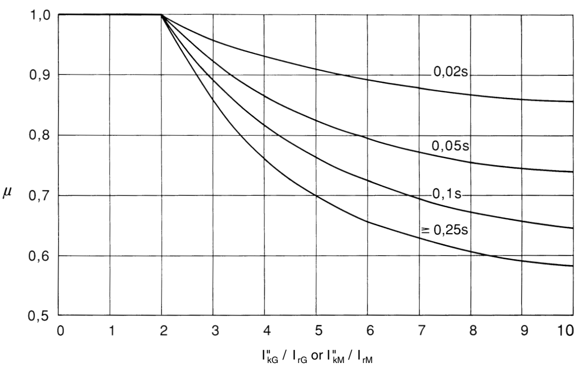


Fig. 3-5

Factor μ for calculating the symmetrical short-circuit breaking current I_a as a function of ratio I''_{kG}/I_{rG} or I''_{kM}/I_{rM}, and of switching delay time t_{min} of 0.02 to 0.25 s.

If the short circuit is fed by a number of independent voltage sources, the symmetrical breaking currents may be added.

With compound excitation or converter excitation one can put $\mu = 1$ if the exact value is not known. With converter excitation Fig. 3-5 applies only if $t_v \leq 0.25$ s and the maximum excitation voltage does not exceed 1.6 times the value at nominal excitation. In all other cases put $\mu = 1$.

The factor q applies to induction motors and takes account of the rapid decay of the motor's short-circuit current owing to the absence of an excitation field. It can be taken from Fig. 3-6 or the equations.

$q = 1.03 + 0.12 \ln m$ for $t_{min} = 0.02$ s

$q = 0.79 + 0.12 \ln m$ for $t_{min} = 0.05$ s

$q = 0.57 + 0.12 \ln m$ for $t_{min} = 0.10$ s

$q = 0.26 + 0.12 \ln m$ for $t_{min} = 0.25$ s

$q_{max} = 1$

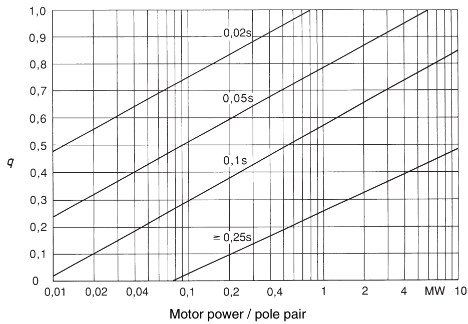


Fig. 3-6

Factor q for calculating the symmetrical short-circuit breaking current of induction motors as a function of the ratio motor power / pole pair and of switching delay time t_{min} of 0.02 to 0.25 s.

Taking account of transformers

The impedances of equipment in the higher- or lower-voltage networks have to be re-calculated with the square of the rated transformer ratio $\ddot{u}_r$ (main tap).

The influence of motors

Synchronous motors and synchronous condensers are treated as synchronous generators.

Induction motors contribute values to I''_k, i_p and I_a and in the case of a two-phase short circuit, to I_k as well.

The heaviest short-circuit currents I''_k, i_p, I_a and I_k in the event of three-phase and two-phase short circuits are calculated as shown in Table 3-3.

For calculating the peak short-circuit current:
κ_m = 1.65 for HV motors, motor power per pole pair < 1MW
κ_m = 1.75 for HV motors, motor power per pole pair ≥ 1MW
κ_m = 1.3 for LV motors

3

Table 3-3

To calculate short-circuit currents of induction motors with terminal short circuit

	three-phase	two-phase
Initial symmetrical short-circuit current	$I''_{k3M} = \dfrac{c \cdot U_n}{\sqrt{3} \cdot Z_M}$	$I''_{k2M} = \dfrac{\sqrt{3}}{2} I''_{k3M}$
Peak short-circuit current	$I''_{p3M} = \kappa_m \sqrt{2}\, I''_{k3M}$	$I''_{p2M} = \dfrac{\sqrt{3}}{2} i_{p3M}$
Symmetrical short-circuit breaking current	$I_{a3M} = I''_{k3M}$	$I''_{a2M} \sim \dfrac{\sqrt{3}}{2} I''_{k3M}$
Steady-state short-circuit current	$I''_{k3M} = 0$	$I_{k2M} \sim \dfrac{1}{2} I''_{k3M}$

The influence of induction motors connected to the faulty network by way of transformers can be disregarded if

$$\frac{\Sigma P_{rM}}{\Sigma S_{rT}} \leqq \frac{0.8}{\dfrac{100\, \Sigma S_{rT}}{S''_k} - 0.3}.$$

Here,

ΣP_{rM} is the sum of the ratings of all high-voltage and such low-voltage motors as need to be considered,

ΣS_{rT} is the sum of the ratings of all transformers feeding these motors and

S''_k is the initial fault power of the network (without the contribution represented by the motors).

To simplify calculation, the rated current I_{rM} of the low-voltage motor group can be taken as the transformer current on the low-voltage side.

%/MVA system

The %/MVA system is particularly useful for calculating short-circuit currents in high-voltage networks. The impedances of individual items of electrical equipment in %/MVA can be determined easily from the characteristics, see Table 3-4.

Table 3-4

Formulae for calculating impedances or reactances in %/MVA

Network component		Impedance z or reactance x	
Synchronous machine	$\frac{x''_d}{S_r}$	x''_d = Subtransient reactance S_r = Rated apparent power	in % in MVA
Transformer	$\frac{u_k}{S_r}$	u_k = Impedance voltage drop S_r = Rated apparent power	in % in MVA
Current-limiting reactor	$\frac{u_r}{S_D}$	u_r = Rated voltage drop S_D = Throughput capacity	in % in MVA
Induction motor	$\frac{I_r/I_{start}}{S_r} \cdot 100\,\%$	I_r = Rated current I_{start} = Starting current (with rated voltage and rotor short-circuited) S_r = Rated apparent power	 in MVA
Line	$\frac{Z' \cdot l \cdot 100\,\%}{U_n^2}$	Z' = Impedance per conductor U_n = Nominal system voltage l = Length of line	in Ω/km in kV in km
Series capacitor	$\frac{-X_c \cdot 100\,\%}{U_n^2}$	X_c = Reactance per phase U_n = Nominal system voltage	in Ω in kV
Shunt capacitor	$-\frac{100\,\%}{S_r}$	S_r = Rated apparent power	in MVA
Network	$\frac{1.1 \cdot 100\,\%}{S''_{kQ}}$	S''_{kQ} = Three-phase initial symmetrical short-circuit power at point of connection Q	in MVA

Table 3-5

Reference values for Z_2/Z_1 and Z_2/Z_0

		Z_2/Z_1	Z_2/Z_0
to calculate			
I''_k	near to generator	1	–
	far from generator	1	–
I_k	near to generator	0.05…0.25	–
	far from generator	0.25…1	–
Networks	with isolated neutral	–	0
	with earth compensation	–	0
	with neutral earthed via impedances	–	0…0.25
Networks with effectively earthed neutral		–	> 0.25

Calculating short-circuit currents by the %/MVA system generally yields sufficiently accurate results. This assumes that the ratios of the transformers are the same as the ratios of the rated system voltages, and also that the nominal voltage of the network components is equal to the nominal system voltage at their locations.

Short-circuit currents with asymmetrical faults

The equations for calculating initial short-circuit currents I''_k are given in Table 3-2.

The kind of fault which produces the highest short-circuit currents at the fault site can be determined with Fig. 3-7. The double earth fault is not included in Fig. 3-7; it results in smaller currents than a two-phase short-circuit. For the case of a two-phase-to-earth fault, the short-circuit current flowing via earth and earthed conductors I''_{kE2E} is not considered in Fig. 3-7.

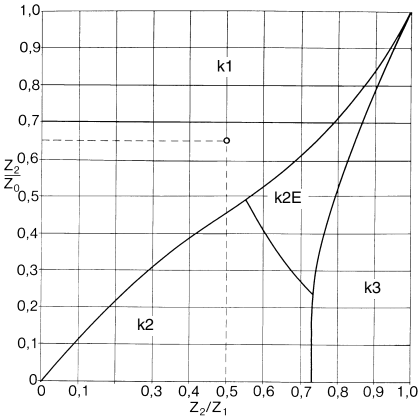


Fig. 3-7

Diagram for determining the fault with the highest short-circuit current

Example: $Z_2/Z_1 = 0.5$; $Z_2/Z_0 = 0.65$, *the greatest short-circuit current occurs with a phase – to-earth fault.*

The data in Fig. 3-7 are true provided that the impedance angles of $\underline{Z}_2/\underline{Z}_1$ and $\underline{Z}_0$ do not differ from each other by more than 15 °. Reference values for Z_2/Z_1 and Z_2/Z_0 are given in Table 3-5.

i_p and I_k are:

for phase-to-phase fault clear of ground: $i_{p2} = \kappa \cdot \sqrt{2} \cdot I''_{k2}$, $I_{k2} = I_{a2} = I''_{k2}$;

for two-phase-to-earth fault: no calculation necessary;

for phase-to-earth fault: $i_{p1} = \kappa \cdot \sqrt{2} \cdot I''_{k1}$, $I_{k1} = I_{a1} = I''_{k1}$.

Fig. 3-8 shows the size of the current with asymmetrical earth faults.

Minimum short-circuit currents

When calculating minimum short-circuit currents one has to make the following changes:

- Reduced voltage factor c
- The network's topology must be chosen so as to yield the minimum short-circuit currents.

– Motors are to be disregarded
– The resistances R_L of the lines must be determined for the conductor temperature t_e at the end of the short circuit (R_{L20} conductor temperature at 20 °C).

$R_L = [1 + 0.004\ (t_e - 20\ °C)/°C] \cdot R_{L20}$

For lines in low-voltage networks it is sufficient to put t_e = 80 °C.

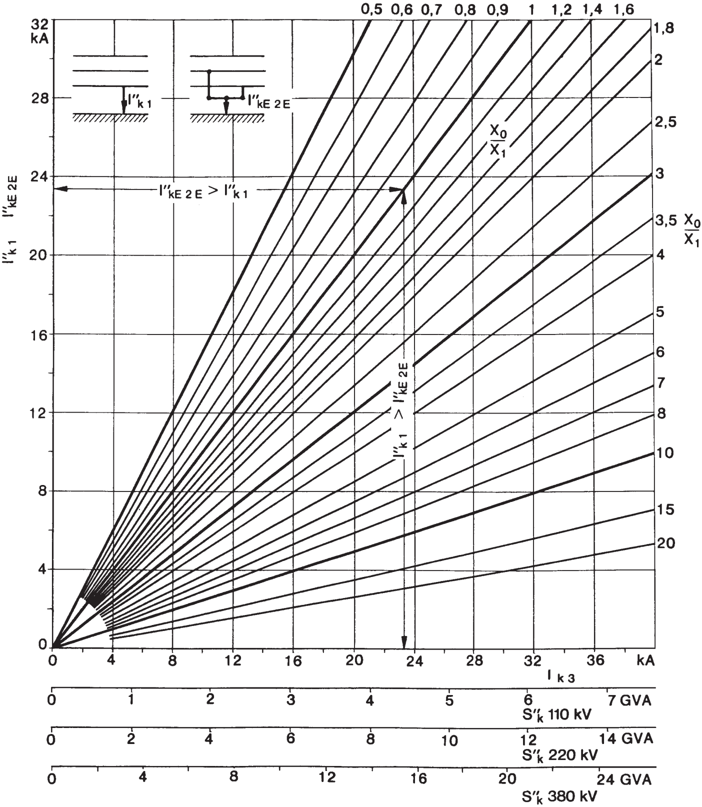


Fig. 3-8

Initial short-circuit current I''_k at the fault location with asymmetrical earth faults in networks with earthed neutral:

S''_k = $\sqrt{3} \cdot UI''_{k3}$ = *Initial symmetrical short-circuit power,*
I''_{kE2E} *Initial short-circuit current via earth for two-phase-to-earth fault,*
I''_{k1} *Initial short-circuit current with phase-to-earth fault,*
X_1, X_0 *Reactances of complete short-circuit path in positive- and zero-phase sequence system ($X_2 = X_1$)*

3.3 Impedances of electrical equipment

The impedances of electrical equipment are generally stated by the manufacturer. The values given here are for guidance only.

3

3.3.1 System infeed

The effective impedance of the system infeed, of which one knows only the initial symmetrical fault power S''_{kQ} or the initial symmetrical short-circuit current I''_{kQ} at junction point Q, is calculated as:

$$Z_Q = \frac{c \cdot U_{nQ}^2}{S''_{kQ}} = \frac{c \cdot U_{nQ}}{\sqrt{3} \cdot I''_{kQ}}$$

Here U_{nQ} Nominal system voltage

S''_{kQ} Initial symmetrical short-circuit power

I''_{kQ} Initial symmetrical short-circuit current

$\underline{Z}_Q = R_Q + jX_Q$, effective impedance of system infeed for short-circuit current calculation

$X_Q = \sqrt{Z_Q^2 - R_Q^2}$.

If no precise value is known for the equivalent active resistance R_Q of the system infeed, one can put $R_Q = 0.1\ X_Q$ *with* $X_Q = 0.995\ Z_Q$. The effect of temperature can be disregarded.

If the impedance is referred to the low-voltage side of the transformer, we have

$$Z_Q = \frac{c \cdot U_{nQ}^2}{S''_{kQ}} \cdot \frac{1}{\ddot{u}_r^2} = \frac{c \cdot U_{nQ}}{\sqrt{3} \cdot I''_{kQ}} \cdot \frac{1}{\ddot{u}_r^2}.$$

3.3.2 Electrical machines

Synchronous generators with direct system connection

For calculating short-circuit currents the positive- and negative-sequence impedances of the generators are taken as

$$\underline{Z}_{GK} = K_G \cdot \underline{Z}_G = K_G\,(R_G + jX''_d)$$

with the correction factor

$$K_G = \frac{U_n}{U_{rg}} \cdot \frac{c_{max}}{1 + X''_d \cdot \sin \varphi_{rg}}$$

Here:

c_{max} Voltage factor

U_n Nominal system voltage

U_{rG} Rated voltage of generator

$\underline{Z}_{GK}$ Corrected impedance of generator

$\underline{Z}_{G}$ Impedance of generator ($\underline{Z}_G = R_G + jX''_d$)

X''_d Subtransient reactance of generator referred to impedance

$$x''_d = X''_d / Z_{rG} \qquad \underline{Z}_{rG} = U^2_{rG}/S_{rG}$$

It is sufficiently accurate to put:

$R_G = 0.05 \cdot X''_d$ for rated powers $\geqq$ 100 MVA } with high-voltage generators

$R_G = 0.07 \cdot X''_d$ for rated powers $<$ 100 MVA } with high-voltage generators

$R_G = 0.15 \cdot X''_d$ for low-voltage generators.

The factors 0.05, 0.07 and 0.15 also take account of the decay of the symmetrical short-circuit current during the first half-cycle.
Guide values for reactances are shown in Table 3-6.

Table 3-6

Reactances of synchronous machines

Generator type	Turbogenerators	Salient-pole generators with damper winding[1]	Salient-pole generators without damper winding
Subtransient reactance (saturated) x''_d in %	9…22[2]	12…30[3]	20…40[3]
Transient reactance (saturated) x''_d in %	14…35[4]	20…45	20…40
Synchronous reactance (unsaturated)[5] x''_d in %	140…300	80…180	80…180
Negative-sequence reactance[6] x_2 in %	9…22	10…25	30…50
Zero-sequence reactance[7] x_0 in %	3…10	5…20	5…25

1) Valid for laminated pole shoes and complete damper winding and also for solid pole shoes with strap connections.

2) Values increase with machine rating. Low values for low-voltage generators.

3) The higher values are for low-speed rotors ($n < 375$ min^{-1}).

4) For very large machines (above 1000 MVA) as much as 40 to 45 %.

5) Saturated values are 5 to 20 % lower.

6) In general $x_2 = 0.5\,(x''_d + x''_q)$. Also valid for transients.

7) Depending on winding pitch.

Generators and unit-connected transformers of power plant units

For the impedance, use

$$\underline{Z}_{G,\,KW} = K_{G,\,KW}\,\underline{Z}_G$$

with the correction factor

$$K_{G,\,KW} = \frac{c_{max}}{1 + X''_d \cdot \sin\varphi_{rG}}$$

$$\underline{Z}_{T,\,KW} = K_{T,\,KW}\,\underline{Z}_{TUS}$$

with the correction factor

$$K_{T,\,KW} = c_{max}.$$

Here:

$\underline{Z}_{G,\,KW}$ $\underline{Z}_{T,\,KW}$	Corrected impedances of generators (G) and unit-connected transformers (T) of power plant units
$\underline{Z}_G$	Impedance of generator
$\underline{Z}_{TUS}$	Impedance of unit transformer, referred to low-voltage side

If necessary, the impedances are converted to the high-voltage side with the fictitious transformation ratio $\ddot{u}_f = U_n/U_{rG}$

Power plant units

For the impedances, use

$$\underline{Z}_{KW} = K_{KW}\,(\ddot{u}_r^2\,\underline{Z}_G + \underline{Z}_{TOS})$$

with the correction factor

$$K_{KW} = \frac{U_{nQ}^2}{U_{rG}^2} \cdot \frac{U_{rTUS}^2}{U_{rTOS}^2} \cdot \frac{c_{max}}{1 + (X''_d - X''_T)\sin\varphi_{rG}}$$

Here:

$\underline{Z}_{KW}$	Corrected impedance of power plant unit, referred to high-voltage side
$\underline{Z}_G$	Impedance of generator
$\underline{Z}_{TOS}$	Impedance of unit transformer, referred to high-voltage side
U_{nQ}	Nominal system voltage
U_{rG}	Rated voltage of generator
X_T	Referred reactance of unit transformer
U_{rT}	Rated voltage of transformer

Synchronous motors

The values for synchronous generators are also valid for synchronous motors and synchronous condensers.

Induction motors

The short-circuit reactance Z_M of induction motors is calculated from the ratio I_{an}/I_{rM}:

$$Z_M = \frac{1}{I_{start}/I_{rM}} \cdot \frac{U_{rM}}{\sqrt{3} \cdot I_{rM}} = \frac{U^2_{rM}}{I_{start}/I_{rM} \cdot S_{rM}}$$

where I_{start} Motor starting current, the rms value of the highest current the motor draws with the rotor locked at rated voltage and rated frequency after transients have decayed,

U_{rM} Rated voltage of motor

I_{rM} Rated current of motor

S_{rM} Apparent power of motor ($\sqrt{3} \cdot U_{rM} \cdot I_{rM}$).

3.3.3 Transformers and reactors

Transformers

Table 3-7

Typical values of impedance voltage drop u_k of three-phase transformers

Rated primary voltage in kV	5…20	30	60	110	220	400
u_k in %	3.5…8	6…9	7…10	9…12	10…14	10…16

Table 3-8

Typical values for ohmic voltage drop u_R of three-phase transformers

Power rating in MVA	0.25	0.63	2.5	6.3	12.5	31.5
u_R in %	1.4…1.7	1.2…1.5	0.9…1.1	0.7… 0.85	0.6…0.7	0.5…0.6

For transformers with ratings over 31.5 MVA, u_R < 0 5 %.

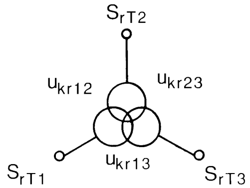
The positive- and negative-sequence transformer impedances are equal. The zero-sequence impedance may differ from this.

The positive-sequence impedances of the transformers $\underline{Z}_1 = \underline{Z}_T = R_T + jX_T$ are calculated as follows:

$$Z_T = \frac{U_{kr}}{100\ \%} \frac{U^2_{rT}}{S_{rT}} \qquad R_T = \frac{u_{Rr}}{100\ \%} \frac{U^2_{rT}}{S_{rT}} \qquad X_T = \sqrt{Z^2_T - R^2_T}$$

With three-winding transformers, the positive-sequence impedances for the corresponding rated throughput capacities referred to voltage U_{rT} are:

a)



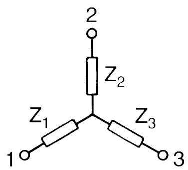
$$|\underline{Z}_{12}| = |\underline{Z}_1| + |\underline{Z}_2| = u_{kr12} \frac{U_{rT}^2}{S_{rT12}}$$

$$|\underline{Z}_{13}| = |\underline{Z}_1| + |\underline{Z}_2| = u_{kr13} \frac{U_{rT}^2}{S_{rT13}}$$

$$|\underline{Z}_{23}| = |\underline{Z}_2| + |\underline{Z}_3| = u_{kr23} \frac{U_{rT}^2}{S_{rT23}}$$

and the impedances of each winding are

b)



$$\underline{Z}_1 = \frac{1}{2}(\underline{Z}_{12} + \underline{Z}_{13} - \underline{Z}_{23})$$

$$\underline{Z}_2 = \frac{1}{2}(\underline{Z}_{12} + \underline{Z}_{23} - \underline{Z}_{13})$$

$$\underline{Z}_3 = \frac{1}{2}(\underline{Z}_{13} + \underline{Z}_{23} - \underline{Z}_{12})$$

Fig. 3-9

Equivalent diagram a) and winding impedance b) of a three-winding transformer
u_{kr12} short-circuit voltage referred to S_{rT12}
u_{kr13} short-circuit voltage referred to S_{rT13}
$u_{kr2}3$ short-circuit voltage referred to S_{rT23}
S_{rT12}, S_{rT13}, S_{rT23} rated throughput capacities of transformer

Three-winding transformers are mostly high-power transformers in which the reactances are much greater than the ohmic resistances. As an approximation, therefore, the impedances can be put equal to the reactances.

The zero-sequence impedance varies according to the construction of the core, the kind of connection and the other windings.

Fig. 3-10 shows examples for measuring the zero-sequence impedances of transformers.

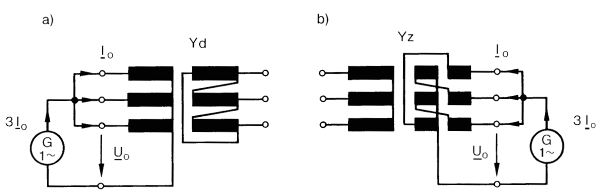
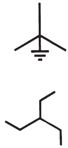
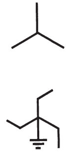


Fig. 3-10

Measurement of the zero-sequence impedances of transformers for purposes of short-circuit current calculation: a) connection Yd, b) connection Yz

Table 3-9

Reference values of X_0/X_1 for three-phase transformers

Connection					
Three-limb core	0.7…1 ∞	3…10 ∞	3…10 ∞	∞ 0.1…0.15	1…2.4 ∞
Five-limb core	1 ∞	10…100 ∞	10…100 ∞	∞ 0,1…0.15	1…2.4 ∞
3 single-phase transformers	1 ∞	10…100 ∞	10…100 ∞	∞ 0,1…0.15	1…2.4 ∞

Values in the upper line when zero voltage applied to upper winding, values in lower line when zero voltage applied to lower winding (see Fig. 3-10).

For low-voltage transformers one can use:

Connection Dy $R_{0T} \approx R_T \quad X_{0T} \approx 0.95\ X_T$

Connection Dz, Yz $R_{0T} \approx 0.4\ R_T \quad X_{0T} \approx 0.1\ X_T$

Connection Yy[1)] $R_{0T} \approx R_T \quad X_{0T} \approx 7…100^{2)}\ X_T$

[1)] Transformers in Yy are not suitable for multiple-earthing protection.
[2)] HV star point not earthed.

Current-limiting reactors

The reactor reactance X_D is

$$X_D = \frac{\Delta u_r \cdot U_n}{100\,\% \cdot \sqrt{3} \cdot I_r} = \frac{\Delta u_r \cdot U_n^2}{100\,\% \cdot S_D}$$

where Δu_r Rated percent voltage drop of reactor

U_n Network voltage

I_r Current rating of reactor

S_D Throughput capacity of reactor.

Standard values for the rated voltage drop

Δu_r in %: 3, 5, 6, 8, 10.

Further aids to calculation are given in Sections 12.1 and 12.2. The effective resistance is negligibly small. The reactances are of equal value in the positive-, negative- and zero-sequence systems.

3.3.4 Three-phase overhead lines

The usual equivalent circuit of an overhead line for network calculation purposes is the Π circuit, which generally includes resistance, inductance and capacitance, Fig. 3-11.

In the positive phase-sequence system, the effective resistance R_L of high-voltage overhead lines is usually negligible compared with the inductive reactance. Only at the low- and medium-voltage level are the two roughly of the same order.

When calculating short-circuit currents, the positive-sequence capacitance is disregarded. In the zero-sequence system, account normally has to be taken of the conductor-earth capacitance. The leakage resistance R_a need not be considered.

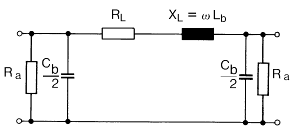


Fig. 3-11

Equivalent circuit of an overhead line

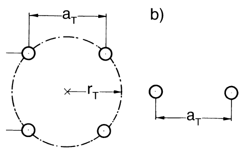


Fig. 3-12

Conductor configurations
a) 4-wire bundle
b) 2-wire bundle

Calculation of positive- and negative-sequence impedance

Symbols used:

a_T Conductor strand spacing,
r Conductor radius,
r_e Equivalent radius for bundle conductors (for single strand $r_e = r$),
n Number of strands in bundle conductor,
r_T Radius of circle passing through midpoints of strands of a bundle (Fig. 3-12),
d Mean geometric distance between the three wires of a three-phase system,
d_{12}, d_{23}, d_{31}, see Fig. 3-13,
r_S Radius of earth wire,
μ_0 Space permeability $4\pi \cdot 10^{-4} \frac{\text{H}}{\text{km}}$,
μ_S Relative permeability of earth wire,
μ_L Relative permeability of conductor (in general $\mu_L = 1$),
ω Angular frequency in s^{-1},
δ Earth current penetration in m,
ρ Specific earth resistance,
R_L Resistance of conductor,
R_S Earth wire resistance (dependent on current for steel wires and wires containing steel),
L_b Inductance per conductor in H/km; $L_b = L_{1.}$

Calculation

The inductive reactance (X_L) for symmetrically twisted single-circuit and double-circuit lines are:

Single-circuit line: $X_L = \omega \cdot L_b = \omega \cdot \frac{\mu_0}{2\pi}\left(\ln \frac{d}{r_e} + \frac{1}{4n}\right)$ in Ω/km per conductor,

Double-circuit line: $X_L = \omega \cdot L_b = \omega \cdot \frac{\mu_0}{2\pi}\left(\ln \frac{d\,d'}{r_e d''} + \frac{1}{4n}\right)$ in Ω/km per conductor;

Mean geometric distances between conductors (see Fig. 3-13):

$$d = \sqrt[3]{d_{12} \cdot d_{23} \cdot d_{31}},$$

$$d' = \sqrt[3]{d'_{12} \cdot d'_{23} \cdot d'_{31}},$$

$$d'' = \sqrt[3]{d''_{11} \cdot d''_{22} \cdot d''_{33}}.$$

The equivalent radius r_e is

$$r_e = \sqrt[n]{n \cdot r \cdot r_T^{n-1}}.$$

In general, if the strands are arranged at a uniform angle *n:*

$$r_e = \frac{a_T}{2 \cdot \sin \frac{\pi}{n}},$$

e. g. for a 4-wire bundle $r_e = \frac{a_T}{2 \cdot \sin \frac{\pi}{4}} = \frac{a_T}{\sqrt{2}}$

The positive- and negative-sequence impedance is calculated as

$$\underline{Z}_1 = \underline{Z}_2 = \frac{R_1}{n} + X_L.$$

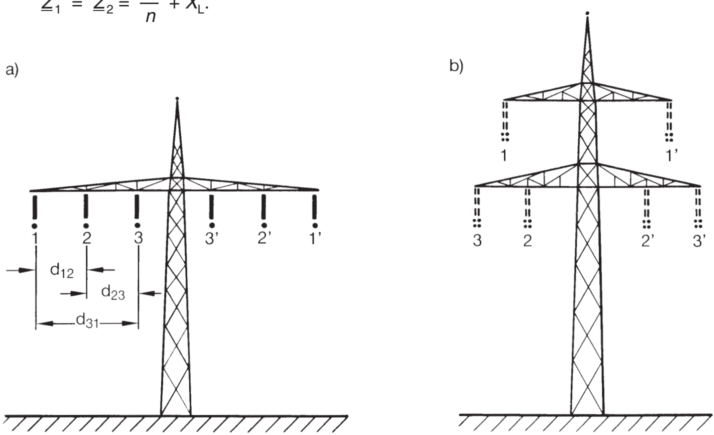


Fig. 3-13

Tower configurations: double-circuit line with one earth wire; a) flat, b) "Donau""

Fig. 3-14 and 3-15 show the positive-sequence (and also negative-sequence) reactances of three-phase overhead lines.

Calculation of zero-sequence impedance

The following formulae apply:

Single-circuit line without earth wire $\underline{Z}_0^{\mathrm{I}} = R_0 + \mathrm{j}X_0,$

Single-circuit line with earth wire $\underline{Z}_0^{\mathrm{Is}} = \underline{Z}_0^{\mathrm{I}} - 3\,\dfrac{\underline{Z}_{\mathrm{as}}^2}{\underline{Z}_{\mathrm{s}}},$

Double-circuit line without earth wire $\underline{Z}_0^{\mathrm{II}} = \underline{Z}_0^{\mathrm{I}} + 3\,\underline{Z}_{\mathrm{ab}},$

Double-circuit line with earth wire $\underline{Z}_0^{\mathrm{IIs}} = \underline{Z}_0^{\mathrm{II}} - 6\,\dfrac{\underline{Z}_{\mathrm{as}}^2}{\underline{Z}_{\mathrm{s}}},$

For the zero-sequence resistance and zero-sequence reactance included in the formulae, we have:

Zero-sequence resistance

$$R_0 = R_{\mathrm{L}} + 3\,\frac{\mu_0}{8}\,\omega, \qquad d = \sqrt[3]{d_{12}\,d_{23}\,d_{31}};$$

Zero-sequence reactance

$$X_0 = \omega\,\frac{\mu_0}{2\pi}\left(3\ln\frac{\delta}{\sqrt[3]{rd^2}} + \frac{\mu_{\mathrm{L}}}{4\,n}\right) \qquad \delta = \frac{1.85}{\sqrt{\mu_0\,\dfrac{1}{\rho}\,\omega}}.$$

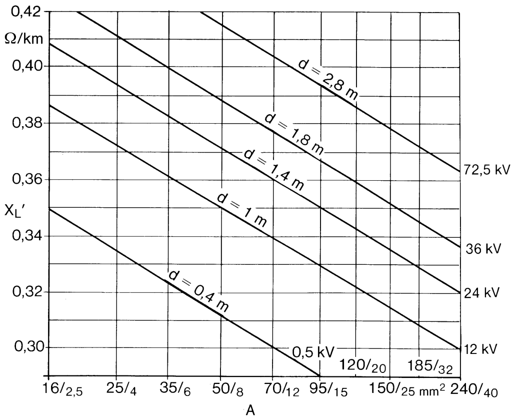


Fig. 3-14

Reactance X_{L}' (positive phase sequence) of three-phase transmission lines up to 72.5 kV, f = 50 Hz, as a function of conductor cross section A, single-circuit lines with aluminium / steel wires, d = mean geometric distance between the 3 wires.

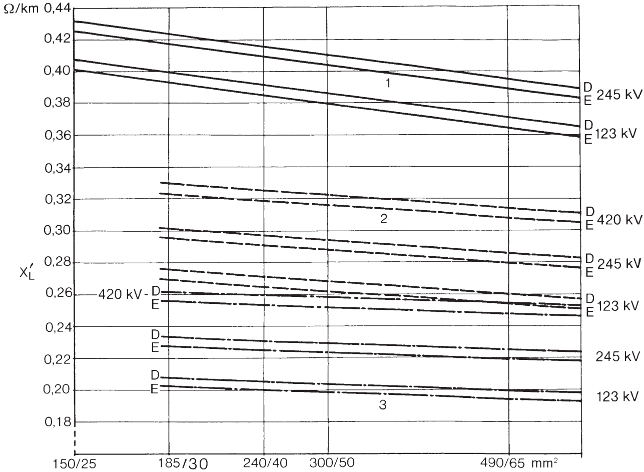


Fig. 3-15

Reactance X'_L (positive-sequence) of three-phase transmission lines with alumimium/steel wires ("Donau" configuration), f = 50 Hz. Calculated for a mean geometric distance between the three conductors of one system, at 123 kV: d = 4 m, at 245 kV: d = 6 m, at 420 kV: d = 9.4 m;
E denotes operation with one system; D denotes operation with two systems; 1 single wire, 2 two-wire bundle, a = 0.4 m, 3 four-wire bundle, a = 0.4 m.

Table 3-10

Earth current penetration δ in relation to specific resistance ρ at f = 50 Hz

Nature of soil as per:	DIN VDE 0228 and CCITT	Alluvial Marl	land Clay	Porous Sandstone, clay schist	Quartz, impervious Limestone	Limestone	Granite, gneiss Clayey slate	
	DIN VDE 0141	Moor-land	—	Loam, clay and soil arable land	Wet sand	Wet gravel	Dry sand or gravel	Stony ground
ρ	Ωm	30	50	100	200	500	1 000	3 000
$\sigma = \frac{1}{\rho}$	μS/cm	333	200	100	50	20	10	3.33
δ	m	510	660	930	1 320	2 080	2 940	5 100

The earth current penetration δ denotes the depth at which the return current diminishes such that its effect is the same as that of the return current distributed over the earth cross section.

Compared with the single-circuit line without earth wire, the double-circuit line without earth wire also includes the additive term $3 \cdot \underline{Z}_{ab}$, where $\underline{Z}_{ab}$ is the alternating impedance of the loops system a/earth and system b/earth:

$$\underline{Z}_{ab} = \frac{\mu_0}{8}\omega + j\,\omega\frac{\mu_0}{2\pi}\ln\frac{\delta}{d_{ab}},$$

$$d_{ab} = \sqrt{d' d''}$$

$$d' = \sqrt[3]{d'_{12} \cdot d'_{23} \cdot d'_{31}},$$
$$d'' = \sqrt[3]{d''_{11} \cdot d''_{22} \cdot d''_{33}}.$$

For a double-circuit line with earth wires (Fig. 3-16) account must also be taken of:

1. Alternating impedance of the loops conductor/earth and earth wire/earth:

$$\underline{Z}_{as} = \frac{\mu_0}{8}\omega + j\,\omega\frac{\mu_0}{2\pi}\ln\frac{\delta}{d_{as}}, \qquad d_{as} = \sqrt[3]{d_{1s}\, d_{2s}\, d_{3s}};$$

for two earth wires:

$$d_{as} = \sqrt[6]{d_{1s1}\, d_{2s1}\, d_{3s1}\, d_{1s2}\, d_{2s2}\, d_{3s2}}$$

2. Impedance of the loop earth wire/earth:

$$\underline{Z}_s = R + \frac{\mu_0}{8}\omega + j\,\omega\frac{\mu_0}{2\pi}\left(\ln\frac{\delta}{r} + \frac{\mu_s}{4\,n}\right).$$

The values used are for one earth wire $n = 1$; $r = r_s$; $R = R_s$;

for two earth wires $n = 2$; $r = \sqrt{r_s\, d_{s1s2}}$; $R = \frac{R_s}{2}$

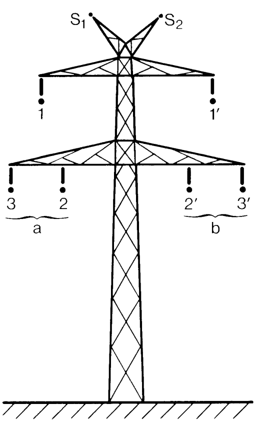


Fig: 3-16

Tower configuration: Double-circuit line with two earth wires, system a and b

Values of the ratio R_s/R_- (effective resistance / d. c. resistance) are roughly between 1.4 and 1.6 for steel earth wires, but from 1.05 to 1.0 for well-conducting earth wires of Al/St, Bz or Cu.

For steel earth wires, one can take an average of $\mu_s \approx 25$, while values of about $\mu_s = 5$ to 10 should be used for Al/St wires with one layer of aluminium. For Al/St earth wires with a cross-section ratio of 6:1 or higher and two layers of aluminium, and also for earth wires or ground connections of Bz or Cu, $\mu_s \approx 1$.

The operating capacitances C_b of high-voltage lines of 110 kV to 380 kV lie within a range of $9 \cdot 10^{-9}$ to $14 \cdot 10^{-9}$ F/km. The values are higher for higher voltages.

The earth wires must be taken into account when calculating the conductor/earth capacitance. The following values are for guidance only:

Flat tower: $C_E = (0.6 \ldots 0.7) \cdot C_b$.

“Donau” tower: $C_E = (0.5 \ldots 0.55) \cdot C_b$

The higher values of C_E are for lines with earth wire, the lower values for those without earth wire.

The value of C_E for double-circuit lines is lower than for single-circuit lines.

The relationship between conductor/conductor capacitance C_g, conductor/earth capacitance C_E and operating capacitance C_b is

$$C_b = C_E + 3 \cdot C_g.$$

Technical values for transmission wires are given in Section 13.1.4.

Table 3-11

Reference values for the impedances of three-phase overhead lines: "Donau" tower, one earth wire, conductor Al/St 240/40, specific earth resistance $\rho = 100\ \Omega \cdot \text{m}$, $f = 50$ Hz

Voltage	d	d_{ab}	d_{as}	Earth wire	Impedance $\underline{Z}_1 = R_1 + \text{j}\,X_1$	Operation with one system zero-sequence impedance $\underline{Z}_0^{\text{I}}$	$\frac{X_0'}{X_1}$	Operation with two systems zero-sequence impedance $\underline{Z}_0^{\text{II}}$	$\frac{X_0''}{X_1}$
	m	m	m		Ω/km per cond.	Ω/km per conductor		Ω/km per cond. and system	
123 kV	4	10	11	St 50	0.12 + j 0.39	0.31 + j 1.38	3.5	0.50 + j 2.20	5.6
				Al/St 44/32		0.32 + j 1.26	3.2	0.52 + j 1.86	4.8
				Al/St 240/40		0.22 + j 1.10	2.8	0.33 + j 1.64	4.2
245 kV	6	15.6	16.5	Al/St 44/32	0.12 + j 0.42	0.30 + j 1.19	2.8	0.49 + j 1.78	4.2
				Al/St 240/40		0.22 + j 1.10	2.6	0.32 + j 1.61	3.8
245 kV 2-wire bundle	6	15.6	16.5	Al/St 240/40	0.06 + j 0.30	0.16 + j 0.98	3.3	0.26 + j 1.49	5.0
420 kV 4-wire bundle	9.4	23	24	Al/St 240/40	0.03 + j 0.26	0.13 + j 0.91	3.5	0.24 + j 1.39	5.3

3.3.5 Three-phase cables

The equivalent diagram of cables can also be represented by Π elements, in the same way as overhead lines (Fig. 3-11). Owing to the smaller spacings, the inductances are smaller, but the capacitances are between one and two orders greater than with overhead lines.

When calculating short-circuit currents the positive-sequence operating capacitance is disregarded. The conductor/earth capacitance is used in the zero phase-sequence system.

Calculation of positive and negative phase-sequence impedance

The a.c. resistance of cables is composed of the d.c. resistance *(R_{-})* and the components due to skin effect and proximity effect. The resistance of metal-clad cables (cable sheath, armour) is further increased by the sheath and armour losses.

The d.c. resistance (R_{-}) at 20 °C and A = conductor cross section in mm² is

for copper: $R'_{-} = \frac{18.5}{A}$ in $\frac{\Omega}{\text{km}}$,

for aluminium: $R'_{-} = \frac{29.4}{A}$ in $\frac{\Omega}{\text{km}}$,

for aluminium alloy: $R'_{-} = \frac{32.3}{A}$ in $\frac{\Omega}{\text{km}}$.

The supplementary resistance of cables with conductor cross-sections of less than 50 mm² can be disregarded (see Section 2, Table 2-8).

The inductance L and inductive reactance X_L at 50 Hz for different types of cable and different voltages are given in Tables 3-13 to 3-17.

For low-voltage cables, the values for positive- and negative-sequence impedances are given in DIN VDE 0102, Part 2 /11.75.

Table 3-12

Reference value for supplementary resistance of different kinds of cable in Ω/km, f = 50 Hz

Type of cable	cross-section mm²	50	70	95	120	150	185	240	300	400
Plastic-insulated cable										
NYCY[1] 0.6/1 kV		—	0.003	0.0045	0.0055	0.007	0.0085	0.0115	0.0135	0.018
NYFGbY[2]	3.5/6 kV to 5.8/10 kV	—	0.008	0.008	0.0085	0.0085	0.009	0.009	0.009	0.009
NYCY[2]	3.5/6 kV to 5.8/10 kV	—	—-	0.0015	0.002	0.0025	0.003	0.004	0.005	0.0065
Armoured lead-covered cable up to 36 kV		0.010	0.011	0.011	0.012	0.012	0.013	0.013	0.014	0.015
Non-armoured aluminium-covered cable up to 12 kV		0.0035	0.0045	0.0055	0.006	0.008	0.010	0.012	0.014	0.018
Non-armoured single-core cable (laid on one plane, 7 cm apart) up to 36 kV										
with lead sheath		0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012
with aluminium sheath		0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
Non-armoured single-core oil-filled cable with lead sheath										
(bundled) 123 kV		—	—	0.009	0.009	0.009	0.0095	0.0095	0.010	0.0105
(laid on one plane, 18 cm apart) 245 kV		—	—	—	—	0.0345	0.035	0.035	0.035	0.035
Three-core oil-filled cable,										
armoured with lead sheath,	36 to 123 kV	0.010	0.011	0.011	0.012	0.012	0.013	0.013	0.014	0.015
non-armoured with	36 kV	—	0.004	0.006	0.007	0.009	0.0105	0.013	0.015	0.018
aluminium sheath,	123 kV	—	—	0.0145	0.0155	0.0165	0.018	0.0205	0.023	0.027

[1] With NYCY 0.6/1 kV effective cross section of C equal to half outer conductor.
[2] With NYFGbY for 7.2/12 kV, at least 6 mm² copper.

Table 3-13

Armoured three-core belted cables[1)], inductive reactance X'_L (positive phase sequence) per conductor at $f = 50$ HZ

Number of cores and conductor cross-section mm²	U = 3.6 kV X'_L Ω/km	U = 7.2 kV X'_L Ω/km	U = 12 kV X'_L Ω/km	U = 17.5 kV X'_L Ω/km	U = 24 kV X'_L Ω/km
3 × 6	0.120	0.144	—	—	—
3 × 10	0.112	0.133	0.142	—	—
3 × 16	0.105	0.123	0.132	0.152	—
3 × 25	0.096	0.111	0.122	0.141	0.151
3 × 35	0.092	0.106	0.112	0.135	0.142
3 × 50	0.089	0.10	0.106	0.122	0.129
3 × 70	0.085	0.096	0.101	0.115	0.122
3 × 95	0.084	0.093	0.098	0.110	0.117
3 × 120	0.082	0.091	0.095	0.107	0.112
3 × 150	0.081	0.088	0.092	0.104	0.109
3 × 185	0.080	0.087	0.09	0.10	0.105
3 × 240	0.079	0.085	0.089	0.097	0.102
3 × 300	0.077	0.083	0.086	—	—
3 × 400	0.076	0.082	—	—	—

1) Non-armoured three-core cables: –15 % of values stated.
Armoured four-core cables: + 10 % of values stated.

Table 3-14

Hochstädter cable (H cable) with metallized paper protection layer, inductive reactance X'_L (positive phase sequence) per conductor at $f = 50$ Hz

Number of cores and conductor cross-section mm²	U = 7.2 kV X'_L Ω/km	U = 12 kV X'_L Ω/km	U = 17.5 kV X'_L Ω/km	U = 24 kV X'_L Ω/km	U = 36 kV X'_L Ω/km
3 × 10 re	0.134	0.143	—	—	—
3 × 16 re or se	0.124	0.132	0.148	—	—
3 × 25 re or se	0.116	0.123	0.138	0.148	—
3 × 35 re or se	0.110	0.118	0.13	0.14	0.154
3 × 25 rm or sm	0.111	0.118	—	—	—
3 × 35 rm or sm	0.106	0.113	—	—	—
3 × 50 rm or sm	0.10	0.107	0.118	0.126	0.138
3 × 70 rm or sm	0.096	0.102	0.111	0.119	0.13
3 × 95 rm or sm	0.093	0.098	0.107	0.113	0.126
3 × 120 rm or sm	0.090	0.094	0.104	0.11	0.121
3 × 150 rm or sm	0.088	0.093	0.10	0.107	0.116
3 × 185 rm or sm	0.086	0.090	0.097	0.104	0.113
3 × 240 rm or sm	0.085	0.088	0.094	0.10	0.108
3 × 300 rm or sm	0.083	0.086	0.093	0.097	0.105

Table 3-15

Armoured SL-type cables[1)], inductive reactance X'_L (positive phase sequence) per conductor at $f = 50$ HZ

Number of cores and conductor cross-section mm²	$U = 7.2$ kV X'_L Ω/km	$U = 12$ kV X'_L Ω/km	$U = 17.5$ kV X'_L Ω/km	$U = 24$ kV X'_L Ω/km	$U = 36$ kV X'_L Ω/km
3 x 6 re	0.171	—	—	—	—
3 x 10 re	0.157	0.165	—	—	—
3 x 16 re	0.146	0.152	0.165	—	—
3 x 25 re	0.136	0.142	0.152	0.16	—
3 x 35 re	0.129	0.134	0.144	0.152	0.165
3 x 35 rm	0.123	0.129	—	—	—
3 x 50 rm	0.116	0.121	0.132	0.138	0.149
3 x 70 rm	0.11	0.115	0.124	0.13	0.141
3 x 95 rm	0.107	0.111	0.119	0.126	0.135
3 x 120 rm	0.103	0.107	0.115	0.121	0.13
3 x 150 rm	0.10	0.104	0.111	0.116	0.126
3 x 185 rm	0.098	0.101	0.108	0.113	0.122
3 x 240 rm	0.096	0.099	0.104	0.108	0.118
3 x 300 rm	0.093	0.096	0.102	0.105	0.113

1) These values also apply to SL-type cables with H-foil over the insulation and for conductors with a high space factor (rm/v and r se/3 f). Non-armoured SL-type cables: –15 % of values stated.

Table 3-16

Cables with XLPE insulation, inductive reactance X'_L (positive phase sequence) per conductor at $f = 50$ Hz, triangular arrangement

Number of cores and conductor cross-section mm²	$U = 12$ kV X'_L Ω/km	$U = 24$ kV X'_L Ω/km	$U = 36$ kV X'_L Ω/km	$U = 72.5$ kV X'_L Ω/km	$U = 123$ kV X'_L Ω/km
3 x 1 x 35 rm	0.135	—	—	—	—
3 x 1 x 50 rm	0.129	0.138	0.148	—	—
3 x 1 x 70 rm	0.123	0.129	0.138	—	—
3 x 1 x 95 rm	0.116	0.123	0.132	—	—
3 x 1 x 120 rm	0.110	0.119	0.126	0.151	0.163
3 x 1 x 150 rm	0.107	0.116	0.123	0.148	0.160
3 x 1 x 185 rm	0.104	0.110	0.119	0.141	0.154
3 x 1 x 240 rm	0.101	0.107	0.113	0.138	0.148
3 x 1 x 300 rm	0.098	0.104	0.110	0.132	0.145
3 x 1 x 400 rm	0.094	0.101	0.107	0.129	0.138
3 x 1 x 500 rm	0.091	0.097	0.104	0.126	0.132
3 x 1 x 630 rm	—	—	—	0.119	0.129

Table 3-17

Cables with XLPE insulation, inductive reactance X'_L (positive phase sequence) per conductor at $f = 50$ Hz

Number of cores and conductor cross-section mm²	$U = 12$ kV X'_L Ω/km
3 x 50 se	0.104
3 x 70 se	0.101
3 x 95 se	0.094
3 x 120 se	0.091
3 x 150 se	0.088
3 x 185 se	0.085
3 x 240 se	0.082

Zero-sequence impedance

It is not possible to give a single formula for calculating the zero-sequence impedance of cables. Sheaths, armour, the soil, pipes and metal structures absorb the neutral currents. The construction of the cable and the nature of the outer sheath and of the armour are important. The influence of these on the zero-sequence impedance is best established by asking the cable manufacturer. Dependable values of the zero-sequence impedance can be obtained only by measurement on cables already installed.

The influence of the return line for the neutral currents on the zero-sequence impedance is particularly strong with small cable cross-sections (less than 70 mm²). If the neutral currents return *exclusively* by way of the neutral (4th) conductor, then

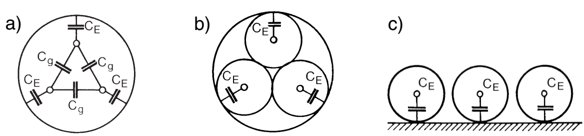
$$R_{0L} = R_L + 3 \cdot R_{\text{neutral}}, \qquad X_{0L} \approx (3{,}5 \ldots 4.0)x_L$$

The zero-sequence impedances of low-voltage cables are given in DIN VDE 0102, Part 2 / 11.75.

Capacitances

The capacitances in cables depend on the type of construction (Fig. 3-17).

With belted cables, the operating capacitance C_b is $C_b = C_E + 3\ C_g$, as for overhead transmission lines. In SL and Hochstädter cables, and with all single-core cables, there is no capacitive coupling between the three conductors; the operating capacitance C_b is thus equal to the conductor/earth capacitance C_E. Fig. 3-18 shows the conductor/earth capacitance C_E of belted three-core cables for service voltages of 1 to 20 kV, as a function of conductor cross-section A. Values of C_E for single-core, SL and H cables are given in Fig. 3-19 for service voltages from 12 to 72.5 kV.



$C_b = C_E = 3\ C_g$ $\quad$ $C_g = 0 \rightarrow C_b = C_E$ $\quad$ $C_g = 0 \rightarrow C_b = C_E$

Fig. 3-17 $\quad C_E \approx 0{,}6\ C_b$

Partial capacitances for different types of cable:
a) Belted cable, b) SL and H type cables, c) Single-core cable

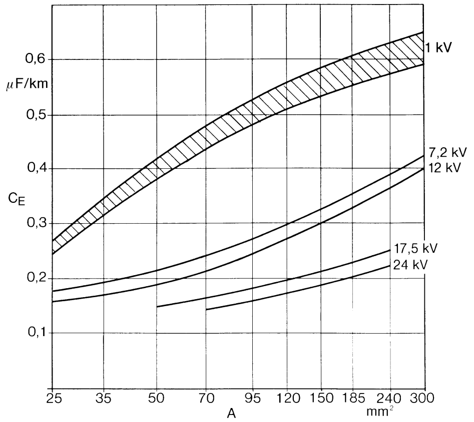


Fig. 3-18

Conductor/earth capacitance C_E of *belted three-core cables as a function of conductor cross-section A. The capacitances of 1 kV cables must be expected to differ considerably.*

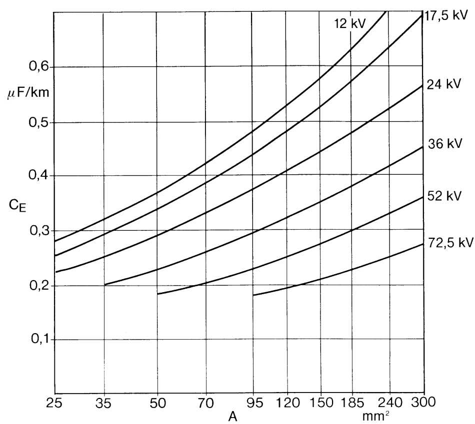


Fig. 3-19

Conductor/earth capacitance C_E of *single-core, SL- and H-type cables as a function of conductor cross-section A.*

The conductor/earth capacitances of XLPE-insulated cables are shown in Tables 3-18 and 3-19.

Table 3-18

Cables with XLPE insulation, conductor/earth capacitance C'_E per conductor

Number of cores and conductor cross-section mm²	U = 12 kV C'_E µF/km	U = 24 kV C'_E µF/km	U = 36 kV C'_E µF/km	U = 72.5 kV C'_E µF/km	U = 123 kV C'_E µF/km
3 x 1 x 35 rm	0.239	—	—	—	—
3 x 1 x 50 rm	0.257	0.184	0.141	—	—
3 x 1 x 70 rm	0.294	0.202	0.159	—	—
3 x 1 x 95 rm	0.331	0.221	0.172	—	—
3 x 1 x 120 rm	0.349	0.239	0.184	0.138	0.110
3 x 1 x 150 rm	0.386	0.257	0.196	0.147	0.115
3 x 1 x 185 rm	0.423	0.285	0.208	0.156	0.125
3 x 1 x 240 rm	0.459	0.312	0.233	0.165	0.135
3 x 1 x 300 rm	0.515	0.340	0.251	0.175	0.145
3 x 1 x 400 rm	0.570	0.377	0.276	0.193	0.155
3 x 1 x 500 rm	0.625	0.413	0.300	0.211	0.165
3 x 1 x 630 rm	—	—	—	0.230	0.185

Table 3-19

Cables with XLPE insulation, conductor/earth capacitance C'_E per conductor

Number of cores and conductor cross-section mm²	U = 12 kV C'_E µF/km
3 x 50 se	0.276
3 x 70 se	0.312
3 x 95 se	0.349
3 x 120 se	0.368
3 x 150 se	0.404
3 x 185 se	0.441
3 x 240 se	0.496

3.3.6 Busbars in switchgear installations

In the case of large cross-sections the resistance can be disregarded.

Average values for the inductance per metre of bus of rectangular section and arranged as shown in Fig. 3-20 can be calculated from

$$L' = 2 \cdot \left[\ln \left(2 \frac{\pi \cdot D + b}{\pi \cdot B + 2\,b} \right) + 0.33 \right] \cdot 10^{-7} \text{ in H/m.}$$

Here:

D Distance between centres of outer main conductor,
b Height of conductor,
B Width of bars of one phase,
L' Inductance of one conductor in H/m.

To simplify calculation, the value for L' for common busbar cross sections and conductor spacings has been calculated per 1 metre of line length and is shown by the curves of Fig. 3-20. Thus,

$$X = 2\,\pi \cdot f \cdot L' \cdot l$$

Example:

Three-phase busbars 40 m long, each conductor comprising three copper bars 80 mm × 10 mm (A = 2400 mm²), distance D = 30 cm, f = 50 Hz. According to the curve, $L' = 3.7 \cdot 10^{-7}$ H/m; and so

$$X = 3.7 \cdot 10^{-7}\ \text{H/m} \cdot 314\ \text{s}^{-1} \cdot 40\ \text{m} = 4.65\ \text{m}\,\Omega.$$

The busbar arrangement has a considerable influence on the inductive resistance.

The inductance per unit length of a three-phase line with its conductors mounted on edge and grouped in phases (Fig. 3-20 and Fig. 13-2a) is relatively high and can be usefully included in calculating the short-circuit current.

Small inductances can be achieved by connecting two or more three-phase systems in parallel. But also conductors in a split phase arrangement (as in Fig. 13-2b) yield very small inductances per unit length of less than 20 % of the values obtained with the method described. With the conductors laid flat side by side (as in the MNS system) the inductances per unit length are about 50 % of the values according to the method of calculation described.

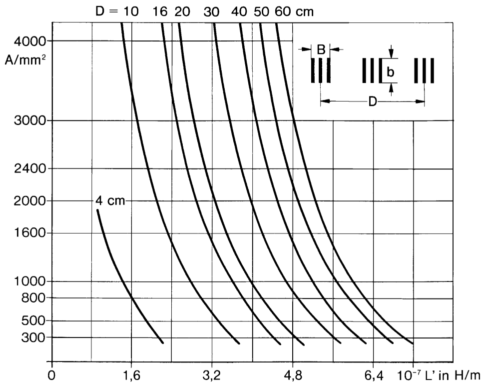


Fig. 3-20

Inductance L´ of busbars of rectangular cross section

3.4 Examples of calculation

More complex phase fault calculations are made with computer programs (Calpos®). See Section 6.1.5 for examples.

When calculating short-circuit currents in high-voltage installations, it is often sufficient to work with reactances because the reactances are generally much greater in magnitude than the effective resistances. Also, if one works only with reactances in the following examples, the calculation is on the safe side. Corrections to the reactances are disregarded.

The ratios of the nominal system voltages are taken as the transformer ratios. Instead of the operating voltages of the faulty network one works with the nominal system voltage. It is assumed that the nominal voltages of the various network components are the same as the nominal system voltage at their respective locations. Calculation is done with the aid of the %/MVA system.

Example 1

To calculate the short-circuit power S_k'', the peak short-circuit current i_p and the symmetrical short-circuit breaking current I_a in a branch of a power plant station service busbar. This example concerns a fault with more than one infeed and partly common current paths. Fig. 3-21 shows the equivalent circuit diagram.

For the reactances of the equivalent circuit the formulae of Table 3-4 give:

Network reactance $$x_Q = \frac{1.1 \cdot 100}{S_{kQ}''} = \frac{110}{8000} = 0.0138\ \%/\text{MVA},$$

Transformer *1* $$x_{T1} = \frac{u_K}{S_{rT1}} = \frac{13}{100} = 0.1300\ \%/\text{MVA},$$

Generator $$x_G = \frac{x_d''}{S_{rG}} = \frac{11.5}{93.7} = 0.1227\ \%/\text{MVA},$$

Transformer *2* $$x_{T2} = \frac{u_K}{S_{rT2}} = \frac{7}{8} = 0.8750\ \%/\text{MVA},$$

Induction motor $$x_{M1} = \frac{I_{rM}/I_{start}}{S_{rM}} \cdot 100 = \frac{1}{5 \cdot 2.69} \cdot 100 = 7.4349\ \%/\text{MVA},$$

Induction-motor group $$x_{M2} = \frac{I_{rM}/I_{start}}{S_{rM}} \cdot 100 = \frac{1}{5 \cdot 8 \cdot 0.46} \cdot 100 = 5.4348\ \%/\text{MVA}.$$

For the location of the fault, one must determine the total reactance of the network. This is done by step-by-step system transformation until there is only one reactance at the terminals of the equivalent voltage source: this is then the short-circuit reactance.

Calculation can be made easier by using Table 3-20, which is particularly suitable for calculating short circuits in unmeshed networks. The Table has 9 columns, the first of which shows the numbers of the lines. The second column is for identifying the parts and components of the network. Columns 3 and 4 are for entering the calculated values.

The reactances entered in column 3 are added in the case of series circuits, while the susceptances in column 4 are added for parallel configurations.

Columns 6 to 9 are for calculating the maximum short-circuit current and the symmetrical breaking current.

To determine the total reactance of the network at the fault location, one first adds the reactances of the 220 kV network and of transformer *1*. The sum 0.1438 %/MVA is in column 3, line 3.

The reactance of the generator is then connected in parallel to this total. This is done by forming the susceptance relating to each reactance and adding the susceptances (column 4, lines 3 and 4).

The sum of the susceptances 15.1041 %/MVA is in column 4, line 5. Taking the reciprocal gives the corresponding reactance 0.0662 %/MVA, entered in column 3, line 5. To this is added the reactance of transformer 2. The sum of 0.9412 %/MVA is in column 3, line 7.

The reactances of the induction motor and of the induction motor group must then be connected in parallel to this total reactance. Again this is done by finding the susceptances and adding them together.

The resultant reactance of the whole network at the site of the fault, 0.7225%/MVA, is shown in column 3, line 10. This value gives

$$S_k'' = \frac{1.1 \cdot 100\ \%}{x_k} \qquad \frac{1.1 \cdot 100\ \%}{0.7225\ \%/\text{MVA}} = 152\ \text{MVA, (column 5, line 10).}$$

3

To calculate the *breaking capacity* one must determine the contributions of the individual infeeds to the short-circuit power S_k''.

The proportions of the short-circuit power supplied via transformer *2* and by the motor group and the single motor are related to the total short-circuit power in the same way as the susceptances of these branches are related to their total susceptance.

Contributions of individual infeeds to the short-circuit power:

Contribution of single motor $\quad S_{kM1}'' = \frac{0.1345}{1.381} \cdot 152 = 14.8$ MVA,

Contribution of motor group $\quad S_{kM2}'' = \frac{0.184}{1.381} \cdot 152 = 20.3$ MVA,

Contribution via transformer *2* $\quad S_{kT2}'' = \frac{1.0625}{1.381} \cdot 152 = 116.9$ MVA.

The proportions contributed by the 220 kV network and the generator are found accordingly.

Contribution of generator $\quad S_{kG}'' = \frac{8.150}{15.104} \cdot 116.9 = 63.1$ MVA,

Contribution of 220 kV network $\quad S_{kQ}'' = \frac{6.954}{15.104} \cdot 116.9 = 53.8$ MVA.

The calculated values are entered in column 5. They are also shown in Fig. 3-21b.

To find the factors μ and q

When the contributions made to the short-circuit power S_k'' by the 220 kV network, the generator and the motors are known, the ratios of S_k''/S_r are found (column 6). The corresponding values of μ for $t_v = 0.1$ s (column 7) are taken from Fig. 3-5.

Values of q (column 8) are obtained from the ratio motor rating / number of pole pairs (Fig. 3-6), again for $t_v = 0.1$ s.

Single motor

$$\frac{S_{kM1}''}{S_{rM1}} = \frac{14.8}{2.69} = 5.50 \rightarrow \mu = 0.74 \qquad \frac{\text{motor rating}}{\text{no. pole pairs}} = \frac{2.3}{2} = 1.15 \rightarrow q = 0.59$$

Motor group

$$\frac{S_{kM2}''}{S_{rM2}} = \frac{20.3}{8 \cdot 0.46} = 5.52 \rightarrow \mu = 0.74 \qquad \frac{\text{motor rating}}{\text{no. pole pairs}} = \frac{0.36}{3} = 1.12 \rightarrow q = 0.32$$

Generator

$$\frac{S_{kG}''}{S_{rG}} = \frac{63.1}{93.7} = 0.67 \rightarrow \mu = 1$$

For the contribution to the short-circuit power provided by the 220 kV network, $\mu = 1$, see Fig. 3-5, since in relation to generator G 3 it is a far-from-generator fault.

Contributions of individual infeeds to the "breaking capacity"

The proportions of the short-circuit power represented by the 220 kV network, the generator and the motors, when multiplied by their respective factors μ and q, yield the contribution of each to the breaking capacity, column 9 of Table 3-20.

Single motor $S_{aM1} = \mu \, q \, S''_{kM1} = 0.74 \cdot 0.59 \cdot 14.8 \text{ MVA} = 6.5 \text{ MVA}$

Motor group $S_{aM2} = \mu \, q \, S''_{kM2} = 0.74 \cdot 0.32 \cdot 20.3 \text{ MVA} = 4.8 \text{ MVA}$

Generator $S_{aG} = \mu \, S''_{kG} = 1 \cdot 63.1 \text{ MVA} = 63.1 \text{ MVA}$

220 kV network $S_{aQ} = \mu \, S''_{kQ} = 1 \cdot 53.8 \text{ MVA} = 53.8 \text{ MVA}$

The total breaking capacity is obtained as an approximation by adding the individual breaking capacities. The result S_a = 128.2 MVA is shown in column 9, line 10.

Table 3-20

Example 1, calculation of short-circuit current

1	2 Component	3 x %/MVA		4 $\frac{1}{x}$ MVA/%	5 S''_k MVA	6 S''_k/S_r	7 μ (0.1 s)	8 q (0.1 s)	9 S_a MVA
1	220 kV network	0.0138		—	53.8	—	1	—	53.8
2	transformer *1*	0.1300		—	—	—	—	—	—
3	1 and 2 in series	0.1438	→	6.9541	—	—	—	—	—
4	93.7 MVA generator	0.1227	→	8.1500	63.1	0.67	1	—	63.1
5	3 and 4 in parallel	0.0662	←	15.1041	—	—	—	—	—
6	transformer *2*	0.8750		—	—	—	—	—	—
7	5 and 6 in series	0.9412	→	1.0625	116.9	—	—	—	—
8	induction motor 2.3 MW/2.69 MVA	7.4349	→	0.1345	14.8	5.50	0.74	0.59	6.5
9	motor group Σ = 3.68 MVA	5.4348	→	0.1840	20.3	5.52	0.74	0.32	4.8
10	fault location 7, 8 and 9 in parallel	0.7225	←	1.3810	152.0	—	—	—	128.2

At the fault location:

$$I''_k = \frac{S''_k}{\sqrt{3} \cdot U_n} = \frac{152.0 \text{ MVA}}{\sqrt{3} \cdot 6.0 \text{ kV}} = 14.63 \text{ kA},$$

$$I_p = \kappa \cdot \sqrt{2} \cdot I''_k = 2.0 \cdot \sqrt{2} \cdot 14.63 \text{ kA} = 41.4 \text{ kA (for } \kappa = 2.0),$$

$$I_a = \frac{S_a}{\sqrt{3} \cdot U_n} = \frac{128.2 \text{ MVA}}{\sqrt{3} \cdot 6.0 \text{ kV}} = 12.3 \text{ kA}.$$

Example 2

Calculation of the phase-to-earth fault current I''_{k1}.

Find I''_{k1} at the 220 kV busbar of the power station represented by Fig. 3-22.

Calculation is made using the method of symmetrical components. First find the positive-, negative- and zero-sequence reactances X_1, X_2 and X_0 from the network data given in the figure.

Positive-sequence reactances (index 1)

Overhead line $$X_{1L} = 50 \cdot 0.32\ \Omega \cdot \frac{1}{2} = 8\ \Omega$$

220 kV network $$X = 0.995 \cdot \frac{1.1 \cdot (220\ \text{kV})^2}{8000\ \text{MVA}} = 6.622\ \Omega$$

Power plant unit $$X_G = 0.14 \cdot \frac{(21\ \text{kV})^2}{125\ \text{MVA}} = 0.494\ \Omega$$

$$X_T = 0.13 \cdot \frac{(220\ \text{kV})^2}{130\ \text{MVA}} = 48.4\ \Omega$$

$$X_{KW} = K_{KW}\,(\ddot{u}_r^2 \cdot X_G + X_T)$$

$$K_{KW} = \frac{1.1}{1 + (0.14 - 0.13) \cdot 0.6}$$

$$X_{KW} = 1.093 \left[\left(\frac{220}{21}\right)^2 \cdot 0.494 + 48.4\right] \Omega = 112.151\ \Omega$$

At the first instant of the short circuit, $x_1 = x_2$. The negative-sequence reactances are thus the same as the positive-sequence values. For the generator voltage: $U_{rG} = 21$ kV with sin $\varphi_{rG} = 0.6$, the rated voltages of the transformers are the same as the system nominal voltages.

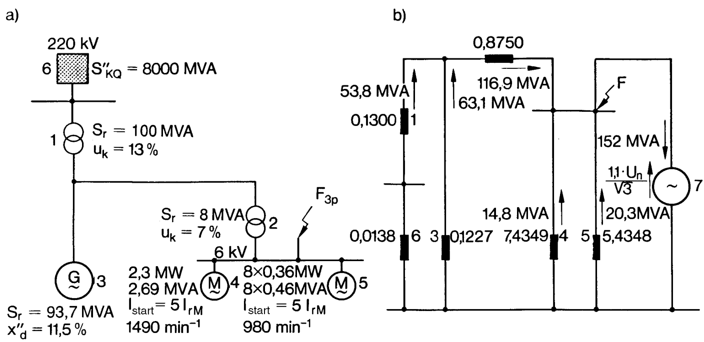


Fig. 3-21

a) Circuit diagram, b) Equivalent circuit diagram in positive phase sequence with equivalent voltage source at fault location, reactances in %/MVA: 1 transformer 1, 2 transformer 2, 3 generator, 4 motor, 5 motor group, 6 220 kV network, 7 equivalent voltage at the point of fault.

Zero-sequence reactances (index 0)

A zero-sequence system exists only between earthed points of the network and the fault location. Generators G1 and G 2 and also transformer T1 do not therefore contribute to the reactances of the zero-sequence system.

Overhead line 2 circuits in parallel	X_{0L}	$= 3.5 \cdot X_{1L} = 28\ \Omega$
220 kV network	X_{0Q}	$= 2.5 \cdot X_{1Q} = 16.555\ \Omega$
Transformer T 2	X_{0T_2}	$= 0.8 \cdot X_{1T} \cdot 1.093 = 42.321\ \Omega$

With the reactances obtained in this way, we can draw the single-phase equivalent diagram to calculate I''_{k1} (Fig. 3-22b).

Since the total positive-sequence reactance at the first instant of the short circuit is the same as the negative-sequence value, it is sufficient to find the total positive and zero sequence reactance.

Calculation of positive-sequence reactance:

$$\frac{1}{x_1} = \frac{1}{56.076\ \Omega} + \frac{1}{14.622\ \Omega} \rightarrow x_1 = 11.598\ \Omega$$

Calculation of zero-sequence reactance:

$$\frac{1}{x_0} = \frac{1}{42.321\ \Omega} + \frac{1}{44.556\ \Omega} \rightarrow x_0 = 21.705\ \Omega$$

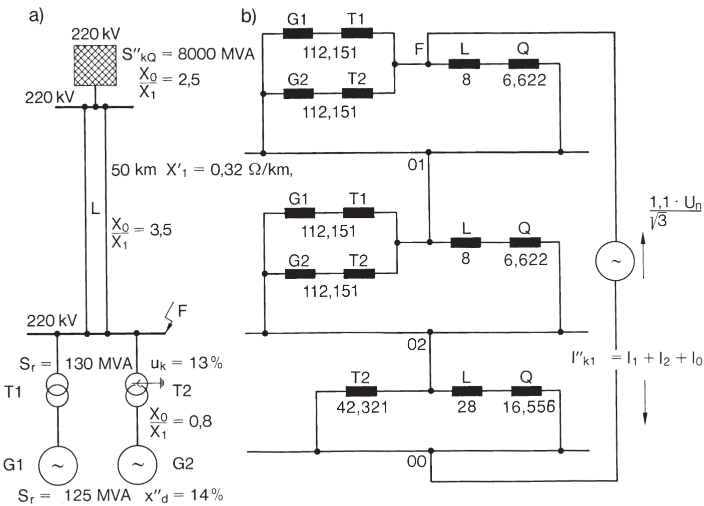


Fig. 3-22

a) Circuit diagram, b) Equivalent circuit diagram in positive phase sequence, negative phase sequence and zero phase sequence with connections and equivalent voltage source at fault location F for I''_{k1}.

With the total positive-, negative- and zero-sequence reactances, we have

$$I''_{k1} = \frac{1.1 \cdot \sqrt{3} \cdot U_n}{x_1 + x_2 + x_0} = \frac{1.1 \cdot \sqrt{3} \cdot 220}{44.901} = 9.34\ \text{kA}.$$

The contributions to I''_{k1} represented by the 220 kV network (Q) or power station (KW) are obtained on the basis of the relationship

$$\underline{I}''_{k1} = \underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 3 \cdot \underline{I}_1 \text{ with } \underline{I}_0 = \underline{I}_1 = \underline{I}_2 = 3.11 \text{ kA}$$

to right and left of the fault location from the equations:

$$\underline{I}''_{k1Q} = \underline{I}_{1Q} + \underline{I}_{2Q} + \underline{I}_{0Q}, \text{ and } \underline{I}''_{k1KW} = \underline{I}_{1KW} + \underline{I}_{2KW} + \underline{I}_{0KW}.$$

3

The partial component currents are obtained from the ratios of the respective impedances.

$$I_{1Q} = I''_{2Q} = 3.11 \text{ kA} \cdot \frac{56.08}{70.70} = 2.47 \text{ kA}$$

$$I_{0Q} = 3.11 \text{ kA} \cdot \frac{42.32}{86.88} = 1.51 \text{ kA}$$

$$I_{1KW} = 0.64 \text{ kA}$$

$$I_{0KW} = 1.60 \text{ kA}$$

$$\underline{I}''_{k1Q} = (2.47 + 2.47 + 1.51) \text{ kA} = 6.45 \text{ kA}$$

$$\underline{I}''_{k1KW} = (0.641 + 0.64 + 1.60) \text{ kA} = 2.88 \text{ kA}$$

Example 3

The short-circuit currents are calculated with the aid of Table 3-2.

20 kV network: $x_{1Q} = 0.995 \dfrac{1.1 \cdot (0.4)^2}{250} = 0.0007\ \Omega$

$r_{1Q} \approx 0.1\ x_{1Q} = 0.00007\ \Omega$

Transformer: $x_{1T} = 0.058 \dfrac{(0.4)^2}{0.63} = 0.0147\ \Omega$

$r_{1T} = 0.015 \dfrac{(0.4)^2}{0.63} = 0.0038\ \Omega$

$x_{0T} = 0.95 \cdot x_{1T} = 0.014\ \Omega$

$r_{0T} \approx r_{1T} = 0.0038\ \Omega$

Cable: $x_{1L} = 0.08 \cdot 0.074 = 0.0059\ \Omega$

$r_{1L20} = 0.08 \cdot 0.271 = 0.0217\ \Omega$

$r_{1L80} = 1.24 \cdot r_{1L20} = 0.0269\ \Omega$

$x_{0L} \approx 7.36 \cdot x_{1L} = 0.0434\ \Omega$

$r_{0L20} \approx 3.97 \cdot r_{1L20} = 0.0861\ \Omega$

$r_{0L80} = 1.24 \cdot r_{0L20} = 0.1068\ \Omega$

Maximum and minimum short-circuit currents at fault location F 1

a. Maximum short-circuit currents

$$\underline{Z}_1 = \underline{Z}_2 = (0.0039 + \text{j}\ 0.0154)\ \Omega; \quad \underline{Z}_0 = (0.0038 + \text{j}\ 0.0140)\ \Omega$$

$$I''_{k3} = \frac{1.0 \cdot 0.4}{\sqrt{3} \cdot 0.0159} \text{ kA} = 14.5 \text{ kA}$$

$$I''_{k2} = \frac{\sqrt{3}}{2} I''_{k3} = 12.6 \text{ kA}$$

$$I''_{k1} = \frac{\sqrt{3} \cdot 1.0 \cdot 0.4}{0.0463} \text{ kA} = 15.0 \text{ kA}.$$

b. Minimum short-circuit currents

The miminum short-circuit currents are calculated with $c = 0.95$.

Maximum and minimum short-circuit currents at fault location F 2

a. Maximum short-circuit currents

$$\underline{Z}_1 = \underline{Z}_2 = (0.0265 + \mathrm{j}\,0.0213)\ \Omega; \quad \underline{Z}_0 = (0.0899 + \mathrm{j}\,0.0574)\ \Omega$$

$$I''_{k3} = \frac{1.0 \cdot 0.4}{\sqrt{3} \cdot 0.0333}\ \text{kA} = 6.9\ \text{kA}$$

$$I''_{k2} = \frac{\sqrt{3}}{2} I''_{k3} = 6.0\ \text{kA}$$

$$I''_{k1} = \frac{\sqrt{3} \cdot 1.0 \cdot 0.4}{0.1729}\ \text{kA} = 4.0\ \text{kA}.$$

b. Minimum short-circuit currents

The minimum short-circuit currents are calculated with $c = 0.95$ and a temperature of 80 °C.

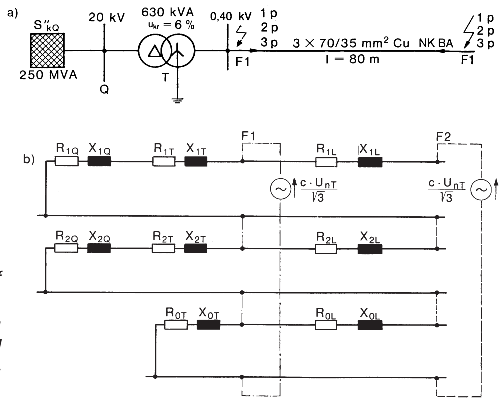


Fig. 3-23

a) Circuit diagram of low-voltage network,
b) Equivalent diagram in component systems and connection for single-phase fault

Table 3-21

Summary of results

Fault location	Max. short-circuit currents 3p kA	2p kA	1p kA	Min. short-circuit currents 3p kA	2p kA	1p kA
Fault location F 1	14.5	12.6	15.0	13.8	12.0	14.3
Fault location F 2	6.9	6.0	4.0	6.4	5.5	3.4

The breaking capacity of the circuit-breakers must be at least 15.0 kA or 6.9 kA. Protective devices must be sure to respond at 12 kA or 3.4 kA. These figures relate to fault location F1 or F2.

3.5 Effect of neutral point arrangement on fault behaviour in three-phase high-voltage networks above 1 kV

Table 3-22

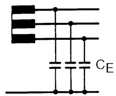
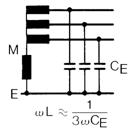
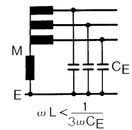
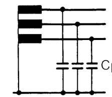
Arrangement of neutral point	isolated	with arc suppression coil	current-limiting *R* or *X*	low-resistance earth
				
Examples of use	Networks of limited extent, power plant auxiliaries	Overhead-line networks 10…123 kV	Cable networks 10…230 kV system e. g. in towns	High-voltage networks (123 kV) to 400 kV (protective multiple earthing in l. v. network)
Between system and earth are:	Capacitances, (inst. transformer inductances)	Capacitances, Suppression coils	Capacitances, Neutral reactor	(Capacitances), Earth conductor
$\lvert \underline{Z}_0/\underline{Z}_1 \rvert$	$\left\lvert \frac{1/\mathrm{j}\omega C_E}{\underline{Z}_1} \right\rvert$	very high resistance	inductive: 4 to 60 resistive: 30 to 60	2 to 4
Current at fault site with single-phase fault Calculation (approximate) $E_1 = \frac{c \cdot U_n}{\sqrt{3}} = E''$	Ground-fault current $\underline{I}_E$ (capacitive) $\underline{I}_E \approx \mathrm{j}\, 3\, \omega C_E \cdot E_1$	Residual ground-fault current $\underline{I}_R$ $\underline{I}_R \approx 3\, \omega C_E(\delta + \mathrm{j}\nu)\, E_1$ δ = loss angle ν = interference	Ground-fault current $\underline{I}_{k1}$ $\underline{I}''_{k1} = \underline{I}_R \approx \frac{3\, E_1}{\mathrm{j}\,(X_1 + X_2 + X_0)}$ $\frac{\underline{I}''_{k1}}{\underline{I}''_{k3}} = \frac{3\, X_1}{2\, X_1 + X_0} = \frac{3}{2 + X_0/X_1}$	(continued)

Table 3-22 (continued)

Arrangement of neutral point	isolated	with arc suppression coil	current-limiting R or X	low-resistance earth
I''_{k2} / I''_{k3}	I_{CE} / I''_{k3}	I_R / I''_{k3}	*inductive*: 0.05 to 0.5 *resistive*: 0.1 to 0.05	0.5 to 0.75
U_{LEmax} / U_n	≈ 1	1 to (1.1)	inductive: 0.8 to 0.95 resistive: 0.1 to 0.05	0.75 to ≦ 0.80
U_{0max} / U_n	≈ 0.6	0.6 to 0.66	inductive: 0.42 to 0.56 resistive: 0.58 to 0.60	0.3 to 0.42
Voltage rise in whole network	yes	yes	no	no
Duration of fault	10 to 60 min Possible short-time earthing with subsequent selective disconnection by neutral current (< 1 s)	10 to 60 min	< 1 s	< 1 s
Ground-fault arc	Self-quenching up to several A	Self-quenching	Partly self-quenching usually sustained	Sustained
Detection	Location by disconnection, ground-fault wiping-contact relay, wattmeter relay. (With short-time earthing: disconnection by neutral current)		Selective disconnection by neutral current (or short-circuit protection)	Short-circuit protection
Risk of double earth fault	yes	yes	slight	no
Means of earthing DIN VDE 0141	Earth electrode voltage $U_E \leq 125$ V Touch voltage ≦ 65 V		Earth electrode voltage $U_E > 125$ V permissible Touch voltages ≦ 65V	
Measures against interference with communication circuits DIN VDE 0228	Generally not necessary needed only with railway block lines	Not necessary	Overhead lines: possibly required if approaching over a considerable distance Cables: generally not necessary	

4 Dimensioning switchgear installations

4.1 Insulation rating

Rating the dielectric withstand of equipment is based on the expected dielectric stresses. This is a combination of the stress caused by the power-frequency continuous voltage and the stress caused by the mostly short-term overvoltages. The insulation coordination for power-frequency continuous voltages ≤ 1 kv is based on DIN VDE 0110 and DIN VDE 0109 (currently still in draft form). In the case of voltages > 1 kV the specifications in DIN EN 60071-1 (VDE 0111 Part l) and the application guide in DIN EN 60071-2 (VDE 0111 Part 2) apply.

The *insulation coordination* is defined in DIN EN 60071-1 (VDE 0111 Part l) as the selection of the dielectric withstand required for equipment that is to be used at a specific site in a network. This process requires knowledge of the operational conditions in the network and the planned overvoltage protection devices, and the probability of an insulation fault on equipment which can be accepted under economic and operational aspects.

The *“dielectric withstand”* can be defined here by a rated insulation level or by a standard insulation level. A rated insulation level is considered any combination of standard withstand voltages, a standard insulation level is considered a rated insulation level whose standard withstand voltages in combination with the associated highest voltage for equipment U_m are recommended in selection tables (Tables 4-1 and 4-2). These combinations are based on operational experience with networks that meet the IEC standard. However, they are not associated with specific operational conditions.

When discussing insulation, a distinction is made between external and internal insulation. *External insulation* consists of clearances in air and the dielectrically stressed surfaces of solid insulation. It is exposed to atmospheric and other effects such as pollution, moisture, animals etc. It can be either protected (indoor) or unprotected (outdoor). The *internal insulation* can be solid, fluid or gaseous insulation material. It is protected against atmospheric and other external effects.

There is also a distinction between *self-restoring and non-self-restoring insulation,* but only with reference to the response of the insulation under dielectric tests. Insulation is considered self-restoring if its insulation properties are restored after a breakdown during the test.

The power frequency voltages and the overvoltages acting on an insulation or an overvoltage protection device can be classified by causes and processes into the following categories:

– power frequency continuous voltages resulting from normal system operation
– temporary overvoltages (power frequency) resulting from earth faults, switching operations (e.g. load shedding, resonances, ferroresonance or similar)
– slow-front overvoltages resulting from switching operations or direct lightning strikes at great distance, with rise times between 20 μs and 5000 μs and times to half-value up to 20 ms

– fast-front overvoltages resulting from switching operations or lightning strikes with rise times between 0.1 μs and 20 μs and times to half-value up to 300 μs
– very fast-front overvoltages resulting from faults or switching operations in gas-insulated switchgear with rise times below 0.1 μs and superimposed oscillations in the frequency range of 30 kHz to 100 MHz with a total duration of 3 ms
– combined overvoltages, primarily between conductors and at open breaker gaps.

It is assumed that within one of these categories the different voltage characteristics can have the same dielectric effects on the insulation or can be converted to a specified characteristic. The following standardized voltage shapes are defined as representative voltage characteristics for the above categories – except for the very fast-front overvoltages:
– standard short-duration power-frequency voltage with a frequency between 48 Hz and 62 Hz and a duration of 60 s
– standard switching impulse voltage; a voltage pulse with a rise time of 250 μs and a time to half-value of 2500 μs
– standard lightning impulse voltage; a voltage pulse with a rise time of 1.2 μs and a time to half-value of 50 μs
– combined standard switching impulse voltage; two simultaneous voltage impulses of opposite polarity

Insulation coordination procedure

The procedure in accordance with DIN EN 60071-1 (VDE 0111 Part l) in its current form requires basic knowledge of the physical processes, the operating conditions and the dielectric response of the equipment with its application. Fig. 4-1 shows the predicted process sequence as a flow chart.

The starting point of the coordination procedure is the system analysis, which should determine what voltage stresses can be expected under operational conditions, possibly with the aid of switching tests in the system. This should also include overvoltage protection devices. The investigations for all ranges of service voltages must include the stress on the conductor-earth insulation, the stress between the conductors and the longitudinal stress on the switching apparatus. The overvoltages must be assessed by peak value, curve and rate of occurrence and classified under the corresponding (curve) categories. The results of the system analysis will include peak values and rate of occurrence of voltage stress in the following categories: short-duration power-frequency voltage, switching impulse voltage, lightning impulse voltage etc. They are shown in the flow chart (Fig. 4-1) as U_{rp}, *representative voltages and overvoltages*.

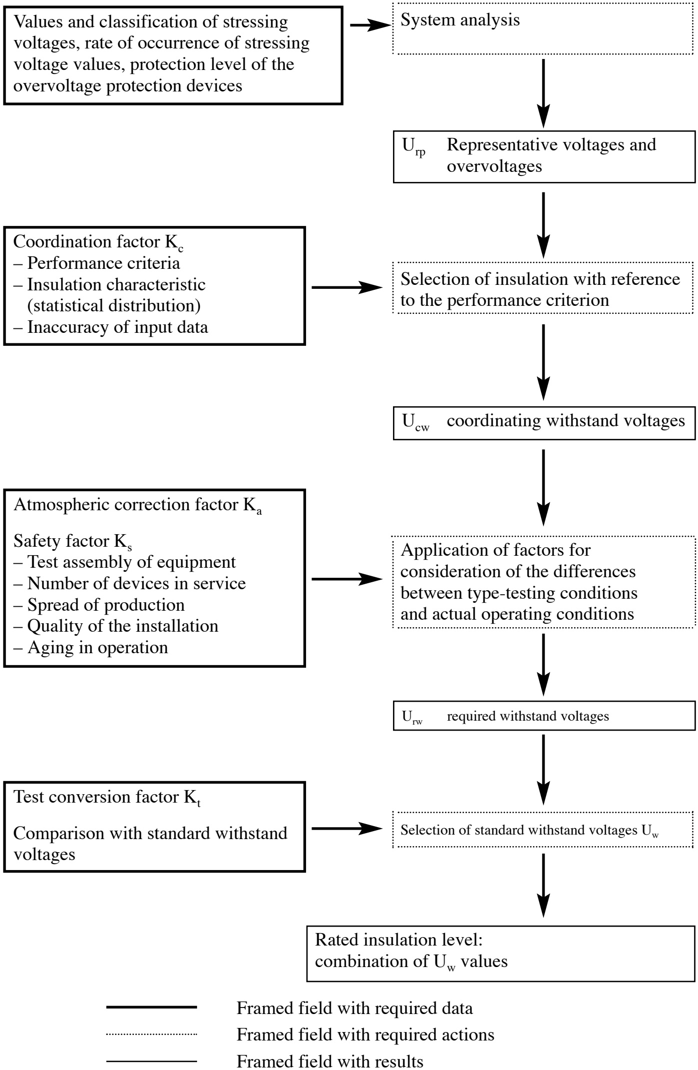


Fig. 4-1

Flow chart for determining the rated insulation level or the standard insulation level

The *performance criterion* is of fundamental importance for the next step. This is given in the form of a permissible fault rate, how often a device at that specific point on the system may be subject to insulation faults caused by the representative voltages and overvoltages (U_{rp}). The next step is to determine the lowest values of the withstand voltages, the equipment must satisfy to meet the Performance criterion. They are referred to as *coordinating withstand voltages* (U_{cw}). The difference between the value of a representative overvoltage and that of the associated coordinating withstand voltage is characterized by the coordination factor K_c, which must be multiplied by the representative overvoltage to derive the coordinating withstand voltage.

To determine the coordination factor K_c with transient overvoltages, a deterministic procedure, a statistical procedure or a combination of the two may be selected. Input quantities are the probability function of the overvoltages (U_{rp}), as the result of the system analysis on one hand and on the other hand, the disruptive discharge probability distribution of the insulation in question. The coordination factor should also include an allowance for any inaccuracies in the input quantities.

The deterministic procedure is used in cases where, for example, with an internal insulation only a conventional withstand voltage (P_w = 100%) can be assumed and this is also protected by a surge arrester. The deterministic layout is also used in the case of overvoltage protection of equipment linked to overhead lines, when the difference between an existing statistical withstand-voltage characteristic (P_w = 90%) and the assumed conventional withstand voltage of the same insulation configuration is taken into consideration by the coordination factor K_c. The deterministic procedure does not leave a defined fault rate for the equipment during operation.

In the statistical procedure, the overvoltage and disruptive discharge probability are available as statistical data and can be combined simultaneously, e.g. with the Monte Carlo method. This calculation must be done for the different kinds of insulation concerned and for different system configurations to determine the total non-availability of a device or an installation.

An insulation can therefore only be economically optimized by statistical design when the downtime expenses are defined for specific fault types. Therefore, the more complex statistical procedure can only be applied in very specific cases, such as the design of switchgear installations for the maximum transmission voltages.

The next step leads from the coordinating withstand voltages (U_{cw}) to the *required withstand voltages* (U_{rw}). Two correction factors are used here. The atmospheric correction factor K_a primarily corrects for the air pressure at the set-up area of the equipment with external insulation, i.e. primarily the altitude. Ambient temperature and humidity have the tendency of acting against each other in their influence on the withstand voltage. The atmospheric conditions generally do not influence the internal insulation.

The atmospheric correction factor is calculated as follows:

$$K_a = e^{m \frac{H}{8150}}$$

H: altitude in metres

m: an exponent that for clean insulators is different from 1 only with switching impulses and that depending on the voltage and geometry of the insulation is to be taken as a guidance value from characteristics (cf. DIN EN 60071-2, Fig. 9!). In the case of contaminated insulators, m is in the range between 0.5 and 0.8 for the power-frequency withstand voltage test.

4

The safety factor K_s considers the number of all other influences that could result in a difference between the equipment in operation and the test object in the type test.

These are:

– aging caused by thermal, dielectric, chemical and mechanical stresses,

– spread caused by manufacturing conditions,

– spread caused by installation, such as changes in the connection technology, parallel loading or numerous devices in operation in comparison to type-testing one single specimen only, etc.

Recommended safety factors are:

– for internal insulation: $K_s = 1.15$,

– for external insulation: $K_s = 1.05$.

If the safety factor of 1.15 applicable for internal insulation is also used for external insulation, the atmospheric correction is also covered to an operational altitude of 1000 m.

The required withstand voltages (U_{rw}) determined to this point are the minimum withstand voltages that must be verified for a device by type tests to ensure that the failure rate predicted in the performance criterion is not exceeded at the operational site in the system. The required withstand voltages can basically be discarded for each of the (curve) categories described above.

The selection tables (Tables 4-1 and 4-2) show standard withstand voltages for the testing of equipment. They show standard voltages for the voltage range I (≤ 245 kV) for testing with short-time power-frequency withstand voltage and with lightning impulse withstand voltage. Voltage range II (> 245 kV) lists standard voltages for testing with lightning impulse withstand voltage and switching impulse withstand voltage.

If the system analysis shows required withstand voltages (U_{rw}) in categories for which the selection tables do not have standard values, conversion to one of the categories listed there is recommended by using corresponding *test conversion factors*. Test conversion factors are listed for the two voltage ranges for internal and external insulation in DIN EN 60071-2 in Tables 2 and 3.

Table 4-1

Standardized insulation levels in voltage range I (1 kV < $U_m \leq$ 245 kV) as per DIN EN 60071-1 (VDE 0111 Part 1)

Highest voltage for equipment U_m kV rms value	Standard short-time power-frequency withstand voltage kV rms value	Standard lightning impulse withstand voltage kV peak value
3.6	10	20 40
7.2	20	40 60
12	28	60 75 95
17.5	38	75 95
24	50	95 125 145
36	70	145 170
52	95	250
72.5	140	325
123	(185) 230	450 550
145	(185) 230 275	(450) 550 650
170	(230) 275 325	(550) 650 750
245	(275) (325) 360 395 460	(650) (750) 850 950 1050

Note: if the values in parentheses are not sufficient to verify that the required conductor-conductor withstand voltages are met, additional withstand voltage tests will be required.

Table 4-2

Standardized insulation levels in range II: U_m > 245 kV
as per DIN EN 60071-1 (VDE 0111 Part 1)

Highest voltage for equipment U_m kV rms value	Standard switching-impulse withstand voltage			Standard lightning impulse withstand voltage kV peak value
	Longitudinal insulation (note 1) kV peak value	Conductor-earth kV peak value	Ratio conductor-conductor to conductor-earth peak value	
300	750	750	1.50	850 950
	750	850	1.50	950 1 050
362	850	850	1.50	950 1 050
	850	950	1.50	1 050 1 175
420	850	850	1.60	1 050 1 175
	950	950	1.50	1 175 1 300
	950	1 050	1.50	1 300 1 425
525	950	950	1.70	1 175 1 300
	950	1 050	1.60	1 300 1 425
	950	1 175	1.50	1 425 1 550
765	1 175	1 300	1.70	1 675 1 800
	1 175	1 425	1.70	1 800 1 950
	1 175	1 550	1.60	1 950 2 100

Note 1: value of the impulse voltage in combined test.

Note 2: the introduction of U_m = 550 kV (instead of 525 kV), 800 kV (instead of 765 kV), 1200 kV and another value between 765 kV and 1200 kV and the associated standard withstand voltages is being considered.

A standardized insulation level from Tables 4-1 and 4-2 must be selected to ensure that in all test voltage categories the values of the required withstand voltages (U_{rw}) are reached or exceeded.

At least two combinations of rated voltage values are assigned to almost every value for the maximum equipment voltage U_m. The result of the procedure for the insulation coordination determines whether the higher or lower values are required, or whether the insulation level of another equipment voltage is to be used.

Note:

The space available here only allows the basics of the (new) procedure for insulation coordination to be considered, but not with all the details. Proper application of the procedure is not trivial; it requires complete familiarity with the material.

This will result in continuing use of the previous procedure in general practice. An exact test will only be economically justifiable with specific projects.

4.2 Dimensioning of power installations for mechanical and thermal short-circuit strength

(as per DIN EN 60865-1 (November 1994), classification VDE 0103, see also IEC 60865-1 (1993-09))[1)]

Symbols used

A	cross section of conductor, with bundle conductors (composite main conductors): total cross- section
a, l or l_s	distances in Fig. 4-2
a_m, a_s	effective main conductor and sub-conductor spacing (Fig. 4-3 and Table 4-3)
a_{12}, $a_{13} \dots a_{1n}$	geometrical distances between the sub-conductors
k_{12}, $k_{13} \dots k_{1n}$	correction factors (Fig. 4-3)
E	Young's modulus
f	operating frequency of the current circuit
f_c	relevant characteristic frequency of a main conductor
F_m or F_s	electrodynamic force between the main or sub-conductors
I_{th}	thermally equivalent short-time current (rms value)
I''_k	initial symmetrical short-circuit current (rms value)
I''_{k2}	initial symmetrical short-circuit current with phase-to-phase short circuit (rms value)
i_p, i_{p2}, i_{p3}	peak short-circuit current or cut-off current of current limiting switchgear or fuses (peak value) with symmetrical short circuit (i_{p2}, i_{p3}: with phase-to-phase or three-phase short circuit)

[1)] see KURWIN calculation program in Table 6-2

J	axial planar moment of inertia (Table 1-22)
m	factor for thermal effect of the d.c. component (Fig. 4-15)
m'	mass per unit length (kg / m) of a conductor without ice, with bundle conductors: total mass per unit length
n	factor for the thermal effect of the a.c. component (Fig. 4-15)
R_{p02}, R'_{p02}	minimum and maximum stress of the yield point (Table 13-1)
S_{thr}	rated short-time current density (rms value) for 1 s
T_k	short-circuit duration
T_{k1}	short-circuit duration with auto-reclosing: duration of the 1st current flow
t	number of sub-conductors
V_r or V_σ	factors for conductor stress
V_F	ratio of dynamic force to static force on the support
V_r	factor for unsuccessful three-phase auto-reclosure in three-phase systems
Z or Z_s	moment of resistance of main or sub-conductor during bending (Table 1-22, shown there with W), also called section modulus as used in DIN EN 60865-1 and in KURWIN
α	factor for force on support (Table 4-4), dependent on the type of busbar and its clamping condition
β	factor for main conductor stress (Table 4-4), dependent on the type of busbar and its clamping condition
γ	factor for determining the relevant characteristic frequency of a conductor (Table 4-4)
κ	factor for calculating the peak short-circuit current i_p as in Fig. 3-1
μ_0	magnetic field constant ($4\pi \cdot 10^{-7}$ H/m)
σ	conductor bending stress

4.2.1 Dimensioning of bar conductors for mechanical short-circuit strength

Parallel conductors whose length l is high in comparison to their distance *a* from one another are subjected to forces evenly distributed along the length of the conductor when current flows. In the event of a short circuit, these forces are particularly high and stress the conductors by bending and the means of fixing by cantilever, pressure or tensile force. This is why busbars must not be designed for the load current only but also to resist the maximum occurring short-circuit current. The load on the busbars and supports to be expected in the event of a short circuit must therefore be calculated. The mechanical short-circuit strength of power installations can also be determined by testing.

The following information is not only applicable to busbars but also to tubular conductors, or very generally to rigid conductors. It is also applicable to two- and three-phase short circuits in a.c. and three-phase systems.

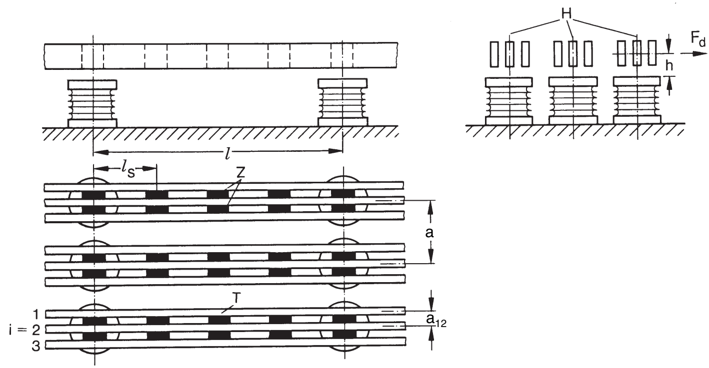


Fig. 4-2

Busbar configuration with three main conductors H with three sub-conductors T each, with spacers Z: a main conductor centre-line spacing, a_{1i} geometrical sub-conductor centre-line spacing clearance (e.g. between the 1st and 2nd sub-conductor a_{12}), F_d support load, h distance between point of application of force and the upper edge of the support, l support distance, l_s maximum distance of a spacer from the support or the adjacent spacer.

IEC 61660-2 applies to calculations in d.c. systems.

When calculating F with three-phase short-circuits for i_p the value $0.93 \cdot i_{p3}$ can be used. The factor 0.93 considers the greatest possible load that can be experienced by the middle conductor of a single-plane configuration in three-phase systems.

The electrodynamic force between the main conductors through which the same current flows is

$$F_m = \frac{\mu_0}{2\pi} \cdot i_p^2 \cdot \frac{l}{a}$$

or as a numerical equation

$$F_m = 0.2 \cdot i_{p2}^2 \cdot \frac{l}{a} \text{ or } F_m = 0.173 \cdot i_{p3}^2 \cdot \frac{l}{a}.$$

If the main conductor consists of t single conductors, the electrodynamic force F_s between the sub-conductors is

$$F_s = \frac{\mu_0}{2\pi} \cdot \left(\frac{i_p}{t}\right)^2 \cdot \frac{l_s}{a_s}$$

or as a numerical equation

$$F_s = 0.2 \cdot \left(\frac{i_p}{t}\right)^2 \cdot \frac{l_s}{a_s}$$

Numerical equations with i_p in kA, F_m in N and l in the same unit as a.

Effective conductor spacing

As previously mentioned, these equations are strictly speaking only for filament-shaped conductors or in the first approximation for conductors of any cross section, so long as their distance from one another is significantly greater than the greatest conductor dimension. If this condition is not met, e.g. with busbar packets comprising rectangular bar conducters, the individual bars must be divided into current filaments and the forces between them calculated. In this case, the actual effective main conductor spacing $a_m = a / k_{1s}$ must be used as the main conductor spacing.

Here, k_{1s} must be taken from Fig. 4-3 where a_{1s} = a and d the total width of the busbar packet in the direction of the short-circuit force. b – as shown in Fig. 4-3 – is the height of the busbars perpendicular to the direction of the short-circuit force.

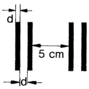
The actual effective sub-conductor clearance is

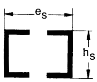
$$\frac{1}{a_s} = \frac{k_{12}}{a_{12}} + \frac{k_{13}}{a_{13}} + \ldots + \frac{k_{1n}}{a_{1n}}$$

For the most frequently used conductor cross sections, a_s is listed in Table 4-3.

Table 4-3

Effective sub-conductor spacing a_s for rectangular cross sections of bars and U-sections (all quantities in cm) as per DIN EN 60865-1 (VDE 0103)

Configuration of bars	Bar thickness *d* cm	Bar width *b* 4 cm	5 cm	6 cm	8 cm	10 cm	12 cm	16 cm	20 cm
	0.5 1	2.0 2.8	2.4 3.1	2.7 3.4	3.3 4.1	4.0 4.7	— 5.4	— 6.7	— 8.0
	0.5 1	— 1.7	1.3 1.9	1.5 2.0	1.8 2.3	2.2 2.7	— 3.0	— 3.7	— 4.3
	1	1.4	1.5	1.6	1.8	2.0	2.2	2.6	3.1
	0.5 1	— 1.74	1.4 1.8	1.5 2.0	1.8 2.2	2.0 2.5	— 2.7	— 3.2	— —

		U 60	U 80	U100	U120	U140	U160	U180	U 200
	h_s =	6	8	10	12	14	16	18	20
	e_s =	8.5	10	10	12	14	16	18	20
	a_s =	7.9	9.4	10	12	14	16	18	20

Stresses on conductors and forces on supports

The bending stress σ of a busbar must not exceed a specified limit in the event of a short circuit to avoid excessive stress on the material. In specifying this limit a sustained bending of the busbar of up to 1 % of the support length has been assumed, because a deformation of this magnitude is virtually undetectable with the naked eye.

The stress on rigid conductors (busbars) and the forces on the supports are influenced by the oscillation response of the conductors. This in return is dependent on the clamping conditions and the permissible plastic deformation or the natural frequency of the conductor. First the upper limit values of the stress are given with consideration to the plastic deformation, while the following section shows the stresses arising from consideration of the oscillation response.

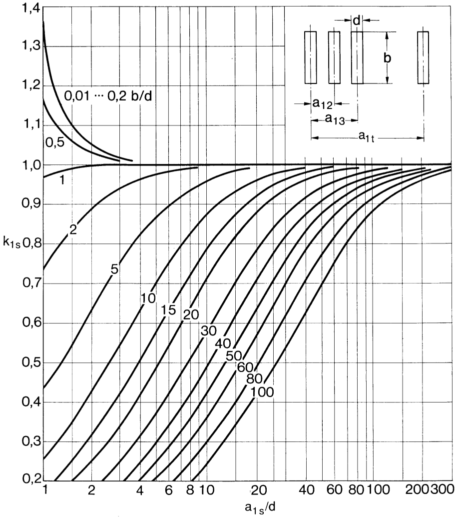


Fig. 4-3

Correction factor k_{1s} for effective main conductor and sub-conductor spacing where s = 2…t

Main conductor stress: $\sigma_m = V_\sigma \cdot V_r \cdot \beta \cdot \dfrac{F_m \cdot l}{8 \cdot Z}$

Sub-conductor stress: $\sigma_s = V_{\sigma s} \cdot V_r \cdot \dfrac{F_s \cdot l_s}{16 \cdot Z_s}$

When considering the plastic deformation

$V_\sigma \cdot V_r = V_{\sigma s} \cdot V_r = 1$ in two-phase a.c. systems

$V_\sigma \cdot V_r = V_{\sigma s} \cdot V_r = 1$ in three-phase systems without three-phase auto-reclosure

$V_\sigma \cdot V_r = V_{\sigma s} \cdot V_r = 1.8$ in three-phase systems with three-phase auto-reclosure

The resulting conductor stress is a combination of the main and sub-conductor stress:

$\sigma_{tot} = \sigma_m + \sigma_s$

The force F_d on each support:

$F_d = V_F \cdot V_r \cdot \alpha \cdot F_m$

with

$V_F \cdot V_r = 1$ for $\sigma_{tot} \geq 0.8 \cdot R'_{p0.2}$

$V_F \cdot V_r = \dfrac{0.8 \cdot R'_{p0.2}}{\sigma_{tot}}$ for $\sigma_{tot} < 0.8 \cdot R'_{p0.2}$

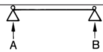
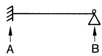
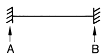
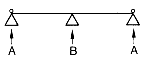
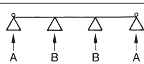
However, in two-phase a.c. systems $V_F \cdot V_r$ does not require a value greater than 2 and in three-phase systems no greater than 2.7.

If it is unclear whether a busbar can be considered supported or fixed at any specific support point, the least suitable case must be taken for rating the busbar and the support.

If the condition $\sigma_{tot} \geq 0.8 \cdot R'_{p0.2}$ is met, the busbar cannot transfer any forces greater than the static forces to the supports, because it will be previously deformed ($V_F \cdot V_r = 1$). However, if σ_{tot} is well below $0.8 \cdot R'_{p0.2}$, it is recommended that conductor and support loads be determined as follows taking into consideration the relevant characteristic frequency of the conductor.

Table 4-4

Factors α, β and γ as per DIN EN 60865-1 (VDE 0103)

Type of busbar and its clamping condition			Force on support Factor α	Main conductor stress Factor β	Relevant charcteristic frequency Factor γ
Single-span beam	both sides supported		*A*: 0.5 *B*: 0,5	1.0	1.57
	fixed, supported		*A*: 0.625 *B*: 0.375	0.73	2.45
	both sides fixed		*A*: 0.5 *B*: 0.5	0.50	3.56
Continuous beam with multiple supprts and *N* equal or approximately equal support distances	$N = 2$		*A*: 0.375 *B*: 1.25	0.73	2.45
	$N \geqq 3$		*A*: 0.4 *B*: 1.1	0.73	3.56

Note to Table 4-4

Continuous beams with multiple supports are continuous bars or tubular conductors that have one or more supports along their length. They are secured against horizontal displacement at one of the supports. The length to be used in the calculation l is the distance between the supports, i.e. the length of the spans, not the length of the continuous beam.

The factors α and β apply for equal support distances. Support distances are still considered equal when the smallest support distance is at least 0.2 times the value of the largest. In this case, end supports are not subject to a higher force than the inner supports. Use the largest support distance for l in the formula.

Stresses on conductors and forces on supports with respect to conductor oscillation

If the characteristic frequency f_c of a conductor is taken into account, lower values for stresses on conductors and forces on supports may be derived than if the characteristic frequency is not considered. If higher values are found here, they are not relevant.

The characteristic frequency of a conductor is

$$f_c = \frac{\gamma}{l^2}\sqrt{\frac{E \cdot J}{m'}}$$

For determining the characteristic frequency of a main conductor, the factor γ is used depending on the clamping conditions in Table 4-4. If the main conductor consists of several sub-conductors, J and m' refer to the main conductor. The data of a sub-conductor should be used for J and m' if there are no stiffening elements along the length of the support distance. In the event that stiffening elements are present, see DIN EN 60865-1 and IEC 60865-1 for additional information. The installation position of the bar conductor with reference to the direction of the short-circuit force must be considered for the axial planar moment of inertia. γ = 3.56 and l for the distance between two stiffening elements must be used for calculating the sub-conductor stresses.

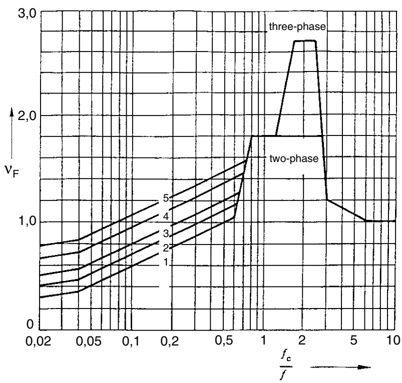


Fig. 4-4
Factor V_F to determine the forces on supports

1: $\kappa \geq 1.60$
2: $\kappa = 1.40$
3: $\kappa = 1.25$
4: $\kappa = 1.10$
5: $\kappa = 1.00$

κ values for Fig. 4-4 and 4-5

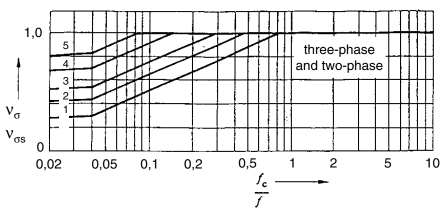


Fig. 4-5

Factors V_σ and $V_{\sigma s}$ to determine the conductor stresses

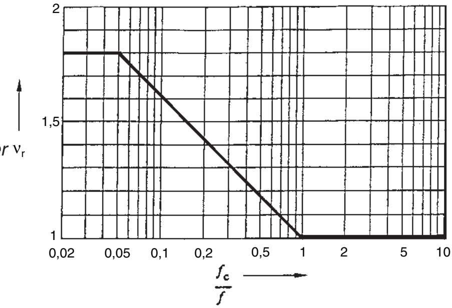
When the characteristic frequencies are considered, the values for V_σ, $V_{\sigma s}$, V_F and V_r to calculate the main conductor and sub-conductor stresses and the forces on supports using the formulae given above may be taken from Fig. 4-4, 4-5 and 4-6 (as per DIN EN 60865-1 (VDE 0103)). At short-circuit durations T_k or T_{k1} of 0.1 s or less the actual stresses and forces may be considerably less than the calculated values with $f_c \leq f$.

With elastic supports the actual value of f_c is less than the calculated value. This needs to be taken into account for $f_c > 2.4\ f$.

Information on digitizing these curves is given in DIN EN 60865-1 and in IEC 60865-1.

Fig. 4-6

Factor V_r, to be used with three-phase auto-reclosing in three-phase systems; in all other cases $V_r = 1$.



Maximum permissible stresses

Conductors are considered short-circuit proof when

$\sigma_{tot} \leq q \cdot R_{p0.2}$ and

$\sigma_s \leq R_{p0.2}$

The plasticity factor q for rectangular busbars is 1.5, for U and I busbars 1.19 or 1.83. Here $q = 1.19$ applies with U busbars with bending around the axis of symmetry of the U, otherwise 1.83. With I busbars $q = 1.83$ applies for bending around the vertical axis of the I, otherwise 1.19. For tubular conductors (with D = external diameter and s = wall thickness) calculate as follows

$$q = 1.7 \cdot \frac{1 - (1 - 2\frac{s}{D})^3}{1 - (1 - 2\frac{s}{D})^4}.$$

The force F_d on the supports must not exceed the minimum breaking force guaranteed by the manufacturer F_r (DIN 48113, DIN EN 60168 – VDE 0674 Part 1) of the insulators. The comparison value for the devices is the rated mechanical terminal load for static + dynamic load. Because this value is not defined in the device standards, it must be obtained from the manufacturer of the devices.

In the case of post insulators that are stressed by cantilever force the distance h of the point of application of force (Fig. 4-2) must be considered.

$F_{red} = k_{red} \cdot F_r$ = reduced rated full load of support.

The reduction factor k_{red} for the approved cantilever force is calculated with the bending moment at the foot of the insulator.

Moments of resistance of composite main conductors

If a stress as in Fig. 4-7a is applied, the main conductor moment of resistance is the sum of the sub-conductor moments of resistance. The same applies for a stress applied as in Fig. 4-7b when there is no or only one stiffening element per span.
Note: The moment of resistance is also called section modulus, as used in DIN EN 60865-1 and in the calculation program KURWIN.

If there are two or more stiffeners, the calculation can be made with higher values for the main conductor moment of resistance. In the case of busbar packets with two or three sub-conductors with a rectangular cross section of 60 %, with more sub-conductors with a rectangular cross section of 50 % and with two or more sub-conductors with a U-shaped cross section of 50 % of the moment of resistance based on the axis 0-0 (ideal) can be used.

If four rectangular sub-conductors are connected in pairs by two or more stiffening elements but there are no stiffening elements between the pairs with the 5 cm spacing, 14 % of the ideal values given in Table 4-5, i.e. $Z_y = 1.73\ b\ d^2$, may be used. The stiffening elements must be installed so that the sub-conductors are prevented from being displaced in a longitudinal direction. The plasticity factor q is exactly as large as that for non-combined main conductors.

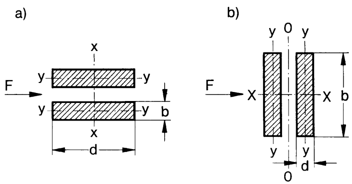
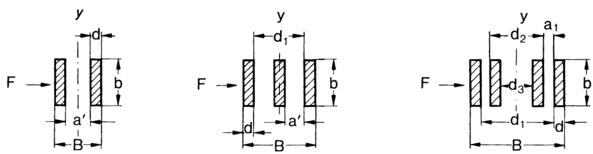


Fig. 4-7

Direction of force and bending axes with conductor packets

Table 4-5

Formulae for calculating the ideal moments of inertia and resistance of composite main conductors with two or more stiffening elements (100 % values).



$J_y = \frac{b}{12}(B^3 - a'^3)$ $\quad J_y = \frac{b}{12}(B^3 - d_1^3 + d^3)$ $\quad J_y = \frac{b}{12}(B^3 - d_1^3 + d_2^3 - d_3^3)$

$Z_y = \frac{b}{6B}(B^3 - a'^3)$ $\quad Z_y = \frac{b}{6B}(B^3 - d_1^3 + d^3)$ $\quad Z_y = \frac{b}{6B}(B^3 - d_1^3 + d_2^3 - d_3^3)$

Cross section mm	J_y cm⁴	Z_y cm³	J_y cm⁴	Z_y cm³	J_y cm⁴	Z_y cm³
Calculated values for J_y in cm⁴ and Z_y in cm³, if $a' = d$ and $d_3 = 5$ cm						
50/5	1.355	1.80	5.15	4.125	—	—
50/10	10.830	7.20	41.25	16.5	341.65	62.10
60/5	1.626	2.16	6.18	4.95	—	—
60/10	12.996	8.64	49.50	19.8	409.98	74.52
80/5	2.168	2.88	8.24	6.60	—	—
80/10	17.328	11.52	66.00	26.4	546.64	99.36
100/5	2.71	3.6	10.3	8.25	—	—
100/10	21.66	14.4	82.5	33	683.3	124.2
120/10	26	17.28	99.00	39.6	819.96	149.04

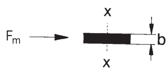
Table 4-6

Moments of inertia and resistance for flat bars

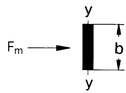
Configuration: flat — upright

Busbar dimensions

flat:



upright:



mm	Z_x cm^3	J_x cm^4	Z_y cm^3	J_y cm^4
12 × 2	0.048	0.0288	0.008	0.0008
15 × 2	0.075	0.0562	0.010	0.001
15 × 3	0.112	0.084	0.022	0.003
20 × 2	0.133	0.133	0.0133	0.00133
20 × 3	0.200	0.200	0.030	0.0045
20 × 5	0.333	0.333	0.083	0.0208
25 × 3	0.312	0.390	0.037	0.005
25 × 5	0.521	0.651	0.104	0.026
30 × 3	0.450	0.675	0.045	0.007
30 × 5	0.750	1.125	0.125	0.031
40 × 3	0.800	1.600	0.060	0.009
40 × 5	1.333	2.666	0.166	0.042
40 × 10	2.666	5.333	0.666	0.333
50 × 5	2.080	5.200	0.208	0.052
50 × 10	4.160	10.400	0.833	0.416
60 × 5	3.000	9.000	0.250	0.063
60 × 10	6.000	18.000	1.000	0.500
80 × 5	5.333	21.330	0.333	0.0833
80 × 10	10.660	42.600	1.333	0.666
100 × 5	8.333	41.660	0.4166	0.104
100 × 10	16.660	83.300	1.666	0.833
120 × 10	24.000	144.000	2.000	1.000
160 × 10	42.600	341.300	2.666	1.333
200 × 10	66.600	666.000	3.333	1.660

Calculation example

Busbar configuration as shown in Fig. 4-2 with three main conductors of three sub-conductors each with rectangular cross section 80 mm × 10 mm of 3.2 m length from

E – Al Mg Si 0.5 F 17.

$R_{p0.2}$ = 12 000 N/cm² (Table 13-1)

$R'_{p0.2}$ = 18 000 N/cm² (Table 13-1)

Stiffeners for each main conductor consist of the tee-off bars and one extra stiffening element in each of the conductors (phases) L1 and L3.

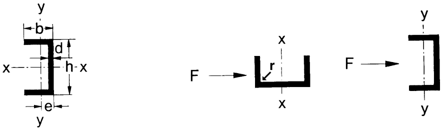
l_s = 40 cm
l = 80 cm
a = 12 cm
a_m = 12.4 cm with k_{1s} = 0.97 as shown in Fig. 4-3 where a_{1s} = a, d = 5 cm, b = 8 cm
a_s = 2.3 cm (Table 4-3)
Z_s = 1.333 cm^3 (Table 4-6)
Z_y = 26.4 cm^3 (Table 4-5)
Z = $0.6 \cdot Z_y = 0.6 \cdot 26.4\ cm^3 = 15.84\ cm^3$
$V_\sigma \cdot V_r = V_{\sigma s} \cdot V_r = 1$
α = 1.1 (Table 4-4 for continuous beam with N ≧ 3, end bay supports α = 0.4)
β = 0.73 (Table 4-4)

Table 4-7

Moments of inertia and resistance for U busbars

Busbar configuration 

U section



Size mm	h mm	b mm	d mm	r mm	e mm	W_x cm^3	J_x cm^4	W_y cm^3	J_y cm^4
50	50	25	4	2	7.71	5.24	13.1	1.20	2.07
60	60	30	4	2	8.96	7.83	23.5	1.76	3.71
70	70	32.5	5	2	9.65	12.4	43.4	2.57	5.87
80	80	37.5	6	2	11.26	19.38	77.5	4.08	10.70
100	100	37.5	8	2	10.96	33.4	167	5.38	14.29
120	120	45	10	3	13.29	59.3	356	9.63	30.53
140	140	52.5	11	3	15.27	90.3	632	14.54	54.15
160	160	60	12	3	17.25	130	1042	20.87	89.22
180	180	67.5	13	3	19.23	180	1622	28.77	138.90
200	200	75	14	3	21.21	241	2414	38.43	206.72

The prospective peak short-circuit current without auto-reclosing is i_{p3} = 90 kA.

$$F_m = 0.173 \cdot i_{p3}^2 \cdot \frac{l}{a_m} = 0.173 \cdot 90^2 \cdot \frac{80}{12.4} = 9041\ \text{N}$$

$$\sigma_m = V_\sigma \cdot V_r \cdot \beta \cdot \frac{F_m \cdot l}{8 \cdot Z} = 1.0 \cdot 0.73 \ \frac{9041\ \text{N} \cdot 80\ \text{cm}}{8 \cdot 15.84\ \text{cm}^3} = 4167\ \text{N/cm}^2$$

$$F_s = 0.2 \left(\frac{i_{p3}}{t}\right)^2 \cdot \frac{l_s}{a_s} = 0.2 \left(\frac{90}{3}\right)^2 \cdot \frac{40}{2.3} = 3130\ \text{N}$$

$$\sigma_s = V_{\sigma s} \cdot V_r \cdot \frac{F_s \cdot l_s}{16 \cdot Z_s} = 1.0 \cdot \frac{3130\ \text{N} \cdot 40\ \text{cm}}{16 \cdot 1.333\ \text{cm}^3} = 5870\ \text{N/cm}^2$$

$$\sigma_{tot} = \sigma_m + \sigma_s = 4\,167 \text{ N/cm}^2 + 5\,870 \text{ N/cm}^2 = 10\,037 \text{ N/cm}^2$$

$$\sigma_{tot} = 10\,037 \text{ N/cm}^2 < 0.8 \cdot R'_{p0.2}$$

$$V_F \cdot V_r = \frac{0.8 \cdot R'_{p0.2}}{\sigma_{tot}} = \frac{0.8 \cdot 18\,000}{10\,037} = 1.44$$

$$F_d = V_F \cdot V_r \cdot \alpha \cdot F_m = 1.44 \cdot 1.1 \cdot 9\,041 = 14\,321 \text{ N}$$

Conductor stresses

$$\sigma_{tot} = 10\,037 \text{ N/cm}^2 < 1.5 \cdot R_{p0,2} = 18000 \text{ N/cm}^2$$

$$\sigma_s = 5\,870 \text{ N/cm}^2 < R_{p0,2} = 12\,000 \text{ N/cm}^2$$

The busbars can be manufactured in accordance with the planned design.

Force on support

If the height of the point of application of force in Fig. 4-2 $h \leq 50$ mm, a post insulator of form C as in Table 13-34 at a rated force F = 16 000 N may be used. If the point of application of the force F is higher than shown in the table, the forces must be converted to take the maximum bending moment at the foot of the insulator into account.

Assessment with respect to the conductor oscillations

Main conductor:
γ = 3.56 (Table 4-4)
l = 80 cm
E = 70 000 N/mm^2 (Table 13-1)
J = $b\,d^3/12$ = 0.67 cm^4 (for single conductors, Table 1-22)
m' = 2.16 kg/m (per sub-conductor, cf. Table 13-7)
f_c = 82.4 Hz (where 1 N = 1 kg m/s^2), valid without stiffening elements
f_c = 144 Hz with stiffening elements (see DIN EN 60865-1)
V_r = 1 (as in Fig. 4-6 where f = 50 Hz and f_c/f = 2.88)
V_σ = 1, V_F = 1.5 (as in Fig. 4-4 and 4-5)
(Regarding the elasticity of the supports, smaller values for f_c must be used, i.e. for V_F with values up to 2.7.)

Sub-conductors:
$\gamma = 3.56$, $l = 40$ cm, $f_{cs} = 330$ Hz, $V_r = 1$, $V_{\sigma s} = 1$

In this case the short, rigid busbars, taking conductor vibrations into account, do not yield smaller values for products $V_\sigma V_r$, $V_{\sigma s} V_r$, $V_F V_r$, i.e. lower stresses than when the plastic deformation is taken into account. This makes the above results determining.

4.2.2 Dimensioning of stranded conductors for mechanical short-circuit strength

The additional electrodynamic force density per unit length F' that a conductor is subjected to with a short circuit is

$$F' = \frac{\mu_0}{2\cdot\pi} \cdot \frac{I''^2_{k2}}{a} \cdot \frac{l_c}{l}$$

where

$$\frac{\mu_0}{2\cdot\pi} = 0.2 \frac{\text{N}}{(\text{kA})^2}.$$

In three-phase systems $I''^2_{k2} = 0.75 \cdot I''^2_{k3}$ must be used.

The length of the span must be used for l and the current-carrying length of the conductor for l_c, i.e. with strained conductors (between portals) the length of the conductor without the length of the string insulators. In the case of slack conductors (inter-equipment connections), $l = l_c$ is the length of the conductor between the equipment terminals.

I''_{k2} and I''_{k3} are the rms values of the initial symmetrical short-circuit current in a two-phase or three-phase short circuit. a is the distance between centres of the main conductors.

Based on this electrodynamic force, the conductors and supports are stressed by the dynamic forces, i.e. by the short-circuit tensile force F_t, the drop force F_f and if applicable by the bundle contraction force (pinch force) F_{pi}. The horizontal span displacement as in Section 4.2.3 must also be considered.

The resulting short-circuit tensile force F_t during the swing out is

with single conductors: $F_t = F_{st} \cdot (1 + \varphi \cdot \psi)$ 1)

with bundle conductors: $F_t = 1{,}1\, F_{st} \cdot (1 + \varphi \cdot \psi)$ 1), 2)

After the short circuit has been tripped, the conductor will oscillate or fall back to its initial state. The maximum value of the conductor pull occurring at the end of the fall, referred to as the drop force F_f, does not need to be considered when the force ratio $r \leq 0.6$ or the maximum swing-out angle is $\delta_m < 70°$.

In all other cases the following applies for the drop force

$$F_f = 1{,}2\, F_{st} \sqrt{1 + 8\zeta \frac{\delta_m}{180°}}$$ 1), 2), 3)

In the case of bundle conductors, if the sub-conductors contract under the influence of the short-circuit current, the tensile force of the bundle conductor will be the bundle contraction force F_{pi}. If the sub-conductors contact one another[4)], i.e. if the parameter $j \geq 1$, F_{pi} is calculated from

$$F_{pi} = F_{st} \left(1 + \frac{\nu_e}{\varepsilon_{st}} \xi\right)$$ 1), 2), 4)

If the sub-conductors do not come into contact during contraction ($j < 1$) F_{pi} is

$$F_{pi} = F_{st} \left(1 + \frac{\nu_e}{\varepsilon_{st}} \eta^2\right)$$ 1), 2)

See page 134 for footnotes

F_{st}[2)], the horizontal component of the static conductor pull, must be taken into account for these calculations[5)], both for the local minimum winter temperature (in Germany usually –20°C) and for the maximum (practical) operating temperature (usually +60°C). The resulting higher values of both tensile forces and and displacement are to be taken into account for the dimensioning. The calculation of the sag from the conductor pull is demonstrated in Sec. 4.3.1. The dependence of the static conductor pull or the conductor tension $\sigma = F_{st}/A$[2)] on the temperature ϑ is derived from

$$\sigma^3 + \left[E \cdot \varepsilon \left(\vartheta - \vartheta_0\right) - \sigma_0 + \frac{E \cdot l^2 \cdot \rho_0^2}{24 \cdot \sigma_0^2} \right] \sigma^2 - \frac{E \cdot l^2}{24} \rho^2 = 0$$

Here σ_0 and ρ_0 values at reference temperature ϑ_0 must be used. ρ_0 is the specific weight, E the practical module of elasticity (Young's modulus) and ε the thermal coefficient of linear expansion of the conductor (see Tables 13-22 ff).

To calculate the short-circuit tensile force:

The load parameter φ is derived from:

$$\varphi = \begin{cases} 3(\sqrt{1+r^2} - 1) & \text{for } T_{k11} \geq T_{res}/4 \\ 3(r \sin\delta_k + \cos\delta_k - 1) & \text{for } T_{k11} < T_{res}/4 \end{cases}$$

T_{k11} = relevant short-circuit duration

T_{k11} = T_{k1} up to a maximum value of 0.4 T

T_{k1} = duration of the first current flow

$$r = \frac{F'}{g_n m'}$$ force ratio [2)]

$$\delta_k = \begin{cases} \delta_1 \left[1 - \cos\left(360° \frac{T_{k11}}{T_{res}} \right) \right] & \text{for } 0 \leq \frac{T_{k11}}{T_{res}} \leq 0{,}5 \\ 2\delta_1 & \text{for } \frac{T_{k11}}{T_{res}} > 0{,}5 \end{cases}$$

Swing-out angle at the end of the short-circuit current flow

1) applicable for horizontal span and horizontal position of wire conductors beside one another, spans to 60 m and sags to 8% of the span length. In the case of larger spans the tensile forces will be calculated as excessive. The calculated tensile force is the horizontal component of the conductor pull and includes the static component.
2) in the case of bundle conductors the values for the complete bundle must be used .
3) in the case of short spans whose length is less than 100 times the diameter of a single conductor, the drop force is calculated too large with this formula because of the stiffness of the conductor.
4) if the sub-conductors are effectively struck together, i.e. clash effectively, it is not necessary to consider F_{pi}. The effective clashing together of the sub-conductors is considered fulfilled if the centre-line distance a_s between two adjacent sub-conductors is equal to or less than x times the conductor diameter d_s and in addition if the distance l_s between two adjacent spacers is at least y times the sub-conductor centre-line distance. x, y can be used as a value pair:
   $x = 2.5$ with $y = 70$
   $x = 2.0$ with $y = 50$
5) see KURWIN calculation program in Table 6-2

$$\delta_1 = \arctan r$$ Direction of the resultant force on the conductor (expressed in degrees)

$$T_{res} = \frac{T}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]}$$ Resultant period of the conductor oscillation

$$T = 2\pi\sqrt{0{,}8\,\frac{b_c}{g_n}}$$ Period of the conductor oscillation

$$b_c = \frac{m'\,g_n\,l^2}{8\,F_{st}}$$ Equivalent static conductor sag in the middle of the span[2)]

Where:

m' mass of a main conductor per unit length[2), 6)]

g_n gravity constant (9.80665 m/s² = 9.80665 N/kg)

The span reaction factor ψ is a function of the stress factor ζ of a main conductor and of the load parameter φ, calculated above, as in Fig. 4-8. It is

$$\zeta = \frac{(g_n\,m'\,l)^2}{24\,F_{st}^3\,N} \quad \text{with} \quad N = \frac{1}{S\,l} + \frac{1}{E_s\,A}$$ Stiffness norm[2)]

Where:

$$E_s = \begin{cases} E\left[0{,}3+0{,}7\sin\left(\frac{F_{st}}{A\sigma_{fin}}90°\right)\right] & \text{for } \frac{F_{st}}{A} \le \sigma_{fin} \\ E & \text{for } \frac{F_{st}}{A} > \sigma_{fin} \end{cases}$$ effective modulus of elasticity[2)]

σ_{fin} 50 N/mm² (Above σ_{fin} the modulus of elasticity is constant .)

E modulus of elasticity (i.e. Young's modulus) of the wire (see Tables 13-22 ff)

S spring constant of the span resulting from elasticity of the supports in the event of short circuit. (For equipment connections S = 100 N/mm, if not otherwise known. In the case of strained conductors between portals, the spring constant must be determined separately. A common value is S = 500 N/mm)

A conductor cross section (actual value or nominal cross section as in Tables 13-24 ff)[2)]

2) see footnote page 134

6) When calculating F_t, F_f and b_h (Sec. 4.2.3) the mass-per-unit length of the main conductor including the distributed single loads must be used.

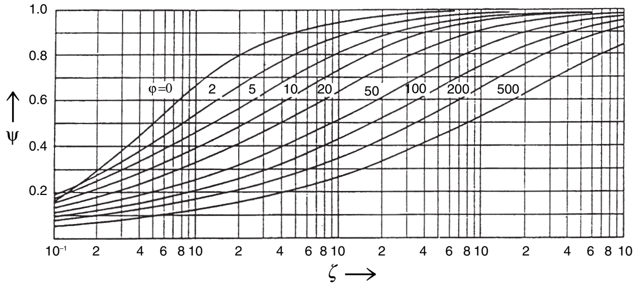


Fig. 4-8

Span reaction factor ψ depending on stress factor ζ and the load parameter φ

Calculating the drop force:

The drop force is particularly dependent on the angle δ_m (see Fig. 4-9) to which the conductor swings out during the short-circuit current flow. Here, for the relevant short-circuit duration T_{k11} must be used as the duration of the short-circuit current T_{k1} (in case of auto-reclosing this is the duration of the first current flow), where the value 0.4 T must be taken as the maximum value for T_{k1} (F_{st} and ζ are given above).

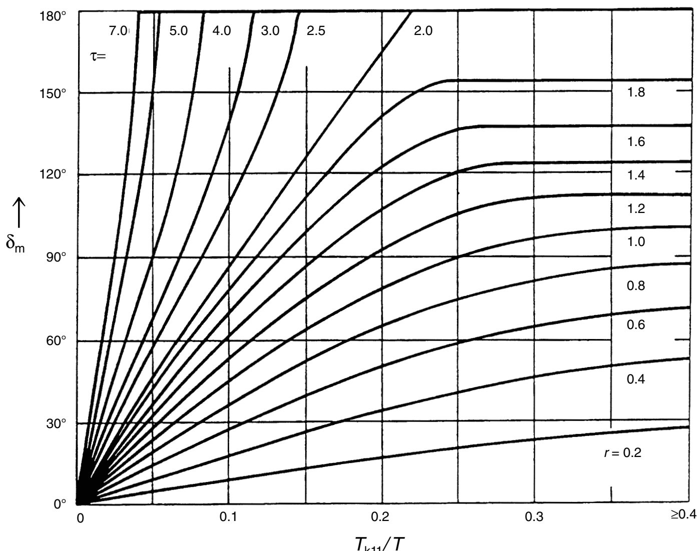


Fig. 4-9

Maximum swing out angle δ_m as function of the relevant short-circuit duration T_{k11} based on the period of the conductor oscillation T

Calculation of the bundle contraction force:

$$j = \sqrt{\frac{\varepsilon_{\mathrm{pi}}}{1+\varepsilon_{\mathrm{st}}}}$$

Parameter for determining the position of the bundle conductor during the short-circuit current flow

$$\varepsilon_{\mathrm{st}} = 1{,}5\,\frac{F_{\mathrm{st}}\,l_{\mathrm{s}}^2\,N}{\left(a_{\mathrm{s}}-d_{\mathrm{s}}\right)^2}\left(\sin\frac{180°}{n}\right)^2$$

Strain factors with bundle conductors

$$\varepsilon_{\mathrm{pi}} = 0{,}375\,n\,\frac{F_{\mathrm{v}}\,l_{\mathrm{s}}^3\,N}{\left(a_{\mathrm{s}}-d_{\mathrm{s}}\right)^3}\left(\sin\frac{180°}{n}\right)^3$$

$$F_{\mathrm{v}} = (n-1)\,\frac{\mu_0}{2\,\pi}\left(\frac{I''_{\mathrm{k}}}{n}\right)^2\frac{l_{\mathrm{s}}}{a_{\mathrm{s}}}\,\frac{\nu_2}{\nu_3}$$

Short-circuit current force between the sub-conductors

- I''_{k} current in the bundle conductor: Maximum value from I''_{k2}, I''_{k3} or I''_{k1}
- I''_{k1} rms value of the initial symmetrical short-circuit current with single-phase short circuit
- n number of sub-conductors of a bundle conductor
- ν_2 see Fig. 4-10 as function of ν_1 and the factor κ
- κ Factor for calculating the peak short-circuit current i_{p} as in Fig. 3-2
- ν_3 see Fig. 4-11 as function of n, a_{s} and d_{s}
- a_{s} centre-line distance between two adjacent sub-conductors
- d_{s} conductor diameter
- l_{s} average distance between two adjacent spacers in a span

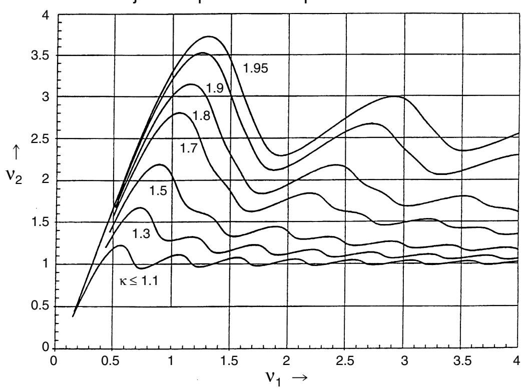


Fig. 4-10
Factor ν_2 as function of ν_1 and κ

$$\nu_1 = f\,\frac{1}{\sin\dfrac{180°}{n}}\sqrt{\frac{\left(a_{\mathrm{s}}-d_{\mathrm{s}}\right)m'_{\mathrm{s}}}{\dfrac{\mu_0}{2\pi}\left(\dfrac{I''_{\mathrm{k}}}{n}\right)^2\dfrac{n-1}{a_{\mathrm{s}}}}}$$

m'_{s} = mass-per-unit length of a sub-conductor
f = frequency of the current circuit

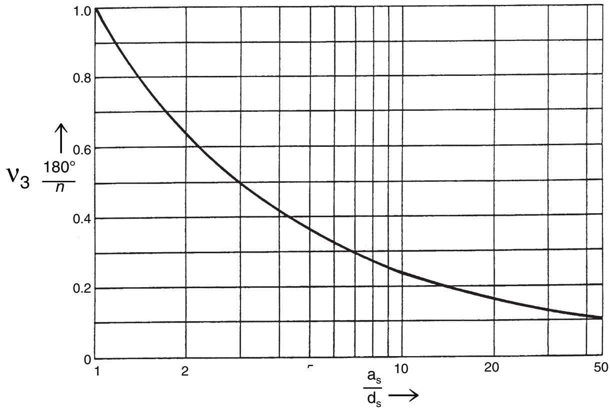


Fig. 4-11

Factor ν_3 as function of the number of sub-conductors n and the bundle dimensions a_s and d_s

Bundle contraction force with sub-conductors in contact, i.e. clashing sub-conductors ($j \geq 1$):

$$\nu_e = \frac{1}{2} + \sqrt{\frac{9}{8}\, n\,(n-1)\,\frac{\mu_0}{2\pi}\left(\frac{I_k''}{n}\right) N\,\nu_2 \left(\frac{l_s}{a_s - d_s}\right)^4 \frac{\left(\sin\frac{180°}{n}\right)^4}{\xi^3}\left(1 - \frac{\arctan\sqrt{\nu_4}}{\sqrt{\nu_4}}\right) - \frac{1}{4}}$$

$$\nu_4 = \frac{a_s - d_s}{d_s}$$

ξ as in Fig. 4-12

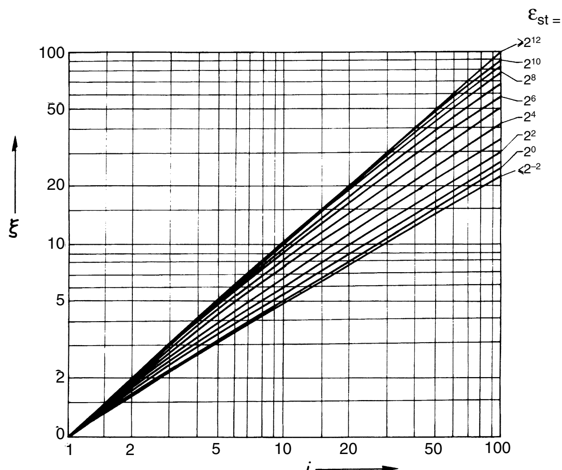


Fig. 4-12

Factor ξ as function of j and ε_{st}

Bundle contraction force with sub-conductors not in contact, i.e. non-clashing sub-conductors ($j < 1$):

$$\nu_e = \frac{1}{2} + \sqrt{\frac{9}{8}\, n\,(n-1)\, \frac{\mu_0}{2\pi} \left(\frac{I_k''}{n}\right) N\, \nu_2 \left(\frac{l_s}{a_s - d_s}\right)^4 \frac{\left(\sin\frac{180^\circ}{n}\right)^4}{\eta^4} \left(1 - \frac{\arctan\sqrt{\nu_4}}{\sqrt{\nu_4}}\right) - \frac{1}{4}}$$

$$\nu_4 = \eta \cdot \frac{a_s - d_s}{a_s - \eta\,(a_s - d_s)}$$

η as in Figs. 4-13a to 4-13c

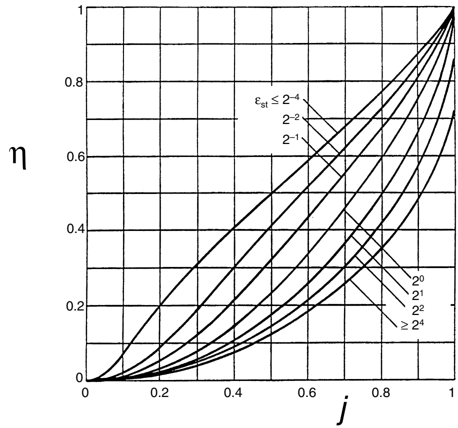


Fig. 4-13a

η as function of j and ε_{st} for $2.5 < a_s / d_s \leq 5.0$

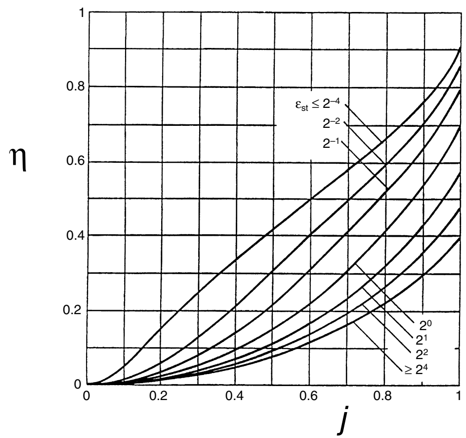


Fig. 4-13b

η as function of j and ε_{st} for $5.0 < a_s / d_s \leq 10.0$

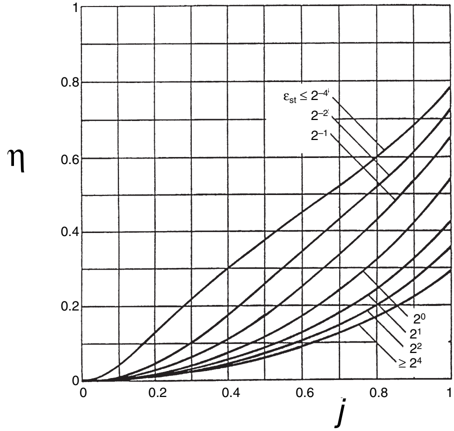


Fig. 4-13c

η as function of j and ε_{st} for $10.0 < a_s / d_s$ {=} 15.0

Permissible loads

For post insulators the maximum value from F_f, F_t and F_{pi} must not exceed the 100% value of the breaking force F_r. For the static load, $F_{st} \leq 0.4\ F_r$ must apply.

For devices the maximum value from F_f, F_t and F_{pi} must not exceed the static + dynamic rated mechanical terminal load. F_{st} may not exceed the (static) rated mechanical terminal load. The conductor clamps must be rated for the maximum value of 1.5 F_t, 1.0 F_f and 1.0 F_{pi}.

For strained conductors, the connectors and supports/portals must be based on the maximum value from F_f, F_t and F_{pi} as a quasi-static exceptional load. Because the loads do not occur at the same time in three-phase configurations, the dynamic force must be assumed as effective in 2 conductors and the static force as effective in the third conductor.

Specifications for rating foundations are in preparation.

Calculation example

Strained conductors between portals in a 420-kV three-phase switchgear installation with current feeder jumpers at the ends and a down-dropper in the middle[7)].

Bundle conductor 2 x Al 1000 mm² as in Tables 13-23 and 13-25

Additional load of the current feeder jumpers and of the down droppers is distributed over the length of the span to the sub-conductors: m'_L = 1.431 kg/m

Centre-line distance of sub-conductors: a_s = 200 mm

Average distance of spacers: l_s = 6.5 m

Span length: l = 42.5 m

Length of bundle conductor between the current feeder jumpers: l_c = 32.5 m

Centre-line distance of main conductors: a = 5 m

Spring constant of the span with static load: S_s = 320.3 N/mm

Spring constant of the span with load caused by short circuit: S_d = 480.5 N/mm

Horizontal static main conductor pull at –20°/60°C: F_{st-20} = 12126.4 N, F_{st+60} = 11370.4 N

Relevant short-circuit current: I''_{k3} = 50 kA, i_p = 125 kA, f = 50 Hz

Short-circuit duration: T_{k1} = 1 s

Calculation of short-circuit tensile force F_t and drop force F_f at –20°C and +60°C

Electrodynamic force density: F' = (0.2 x 0.75 x 50² / 5) (32.5 / 42.5) N/m = 57.35 N/m

Relevant mass of conductor per unit length incl. additional loads:

m' = 2 (2.767 + 1.431) kg/m = 8.396 kg/m

Force ratio: r = 57.35 / (9.80665 x 8.396) = 0.697

Direction of resultant force on the conductor: δ_1 = arctan 0.697 = 34.9°

	–20°C	60°C	
Equivalent static conductor sag b_c	1.53	1.63	m
Period of conductor oscillation T	2.22	2.29	s
Resultant period of oscillation T_{res}	2.06	2.13	s
Relevant short-circuit duration T_{k11}	0.89	0.92	s
Swing-out angle δ_k (with $T_{k11} \le 0.5\ T_{res}$)	66.5	66.5	°
Load parameter φ (with $T_{k11} \ge T_{res}/4$)	0.656	0.656	
Effective modulus of elasticity E_s (with $F_{st}/A \le \sigma_{fin}$)	23791	23342	N/mm²
Stiffness norm N	70	70	10^{-9}/N
Stress factor ζ	4.1	4.9	
Span reaction factor ψ (as in Fig. 4-8)	0.845	0.866	
Short-circuit tensile force F_t (with bundle conductors)	20730	19614	N
Maximum swing-out angle δ_m (as in Fig. 4-9)	79	79	°
Drop force F_f (because r > 0.6 and $\delta_m \ge 70°$)	56961	58326	N

The maximum value of the short-circuit tensile force is derived at the lower temperature and is F_t = 20730 N. The maximum value of the drop force is derived at the higher temperature and is F_f = 58623 N.

7) The calculation was conducted with the KURWIN calculation program (see Table 6-2). This yields more accurate figures than would be possible with manual calculation and would be required with regard to the general accuracy of the procedure.

Calculation of the bundle contraction force F_{pi} at –20°C and +60°C

The contraction force must be calculated because the sub-conductors do not clash effectively. It is $x = a_s/d_s$ = 200 mm / 41.1 mm = 4.87 and $y = l_s / a_s$ = 6.5 m / 0.2 m = 32.5. The condition $y \geq 50$ and $x \leq 2.0$ is not met.

The question whether the sub-conductors come into contact with one another during the contraction is decided at the parameter j as follows:

The relevant short-circuit current is the three-phase short-circuit current (50 kA). The relevant weight of the bundle conductor is only the weight of the two conductors of m' = 2 x 2.767 kg/m = 5.534 kg/m. At a circuit frequency of 50 Hz, this yields the determining parameter v_1 to 1.33.

With factor $\kappa = i_p / \sqrt{2}\, I''_{k3}$ = 125 / (1.41 x 50) = 1.77 factor ν_2 = 2.64 is derived from Fig. 4-10. Fig. 4-11 yields ν_3 = 0.37. These factors yield the short-circuit force between the sub-conductors as F_v = 0.2 25² (6.5 / 0.2) (2.64 / 0.37) N = 29205 N. This gives the following for the two relevant temperatures:

	–20°C	60°C
Strain factor ε_{st}	2.13	2.01
Strain factor ε_{pi}	104.9	105.5
Parameter j	5.79	5.92

Therefore, the sub-conductors do come into contact with one another. This continues as follows:

	–20°C	60°C	
Parameter ξ (as in Fig. 4-12)	4.10	4.14	
Parameter ν_e (at $j \geq 1$)	1.32	1.31	
Bundle contraction force F_{pi}	43032	42092	N

The maximum value of the contraction force occurs at the lower temperature and is F_{pi} = 43032 N.

4.2.3 Horizontal span displacement

The electrodynamic force occurring with short circuits moves the conductors outwards. Depending on the interplay of conductor weight and duration and magnitude of the short-circuit current, a conductor can oscillate completely upwards, then to the other side and again to the bottom of the oscillation, in other words travelling in a complete circle. Furthermore, the conductor is stretched (factor C_D) and the conductor curve is deformed (factor C_F), with the result that a conductor can swing further outwards than would be predicted from its static sag.

The maximum horizontal span displacement b_h (outwards and inwards) in the middle of the span is calculated with slack conductors ($l_c = l$)

$$b_h = \begin{cases} C_F C_D b_c & \text{for} \quad \delta_m \geq 90° \\ C_F C_D b_c \sin\delta_m & \text{for} \quad \delta_m < 90° \end{cases} \quad \text{for} \quad l_c = l$$

and with strained conductors, which are attached to support structures by insulator strings (length l_i).

$$b_h = \begin{cases} C_F C_D b_c \sin\delta_1 & \text{for} \quad \delta_m \geq \delta_1 \\ C_F C_D b_c \sin\delta_m & \text{for} \quad \delta_m < \delta_1 \end{cases} \quad \text{for} \quad l_c = l - 2\, l_i$$

Here, δ_1, b_c and δ_m have the same values, as calculated in Sec. 4.2.2 or as in Fig. 4-9. In three-phase systems the three-phase short-circuit current as in Sec. 4.2.2 must also be used. In addition, the following applies:

$$C_F = \begin{cases} 1{,}05 & \text{for } r \le 0{,}8 \\ 0{,}97 + 0{,}1\, r & \text{for } 0{,}8 < r < 1{,}8 \\ 1{,}15 & \text{for } r \ge 1{,}8 \end{cases}$$ with the force ratio r as in Sec. 4.2.2

$$C_D = \sqrt{1 + \frac{3}{8}\left(\frac{l}{b_c}\right)^2 (\varepsilon_{ela} + \varepsilon_{th})}$$

$$\varepsilon_{ela} = N\left(F_t - F_{st}\right)$$ Elastic conductor expansion

$$\varepsilon_{th} = \begin{cases} c_{th}\left(\dfrac{I''_k}{A}\right)^2 \dfrac{T_{res}}{4} & \text{for } T_{k11} \ge \dfrac{T_{res}}{4} \\ c_{th}\left(\dfrac{I''_k}{A}\right)^2 T_{k1} & \text{for } T_{k11} < \dfrac{T_{res}}{4} \end{cases}$$ Thermal conductor expansion

$$c_{th} = \begin{cases} 0{,}27 \cdot 10^{-18}\ \dfrac{m^4}{A^2 s} & \text{with conductor of Al, AlMgSi, Al/St with cross section-ratio} < 6 \text{ (see Table 13-26)} \\ 0{,}17 \cdot 10^{-18}\ \dfrac{m^4}{A^2 s} & \text{with conductors of Al/St with cross-section ratio} \ge 6 \\ 0{,}088 \cdot 10^{-18}\ \dfrac{m^4}{A^2 s} & \text{with conductors of copper} \end{cases}$$

$I''_k = I''_{k3}$ in three-phase systems or $I''_k = I''_{k2}$ in two-phase a.c. systems

Permissible displacement

In the most unsuitable case two adjacent cables approach each other by the horizontal span displacement b_h. This leaves a minimum distance $a_{min} = a - 2\, b_h$ between them. This minimum distance is reached only briefly during the conductor oscillations. If a subsequent flashover, e.g. at the busbar, is not to occur in the case of a short circuit at some other place, e.g. at a feeder of the switchgear installation, then a_{min} (as per VDE 0101 and HD 637 S1) - of the busbar - must not be less than 50% of the otherwise required minimum distance of conductor – conductor as in Table 4-10.

Calculation example

Strained conductors between portals as in Sec. 4.2.2

To determine the elastic conductor expansion, the short-circuit tensile force also at the upper temperature (60°C) must be known. It was calculated in Sec. 4.2.2. Then

	–20°C	60°C	
Factor for the elastic conductor expansion ε_{ela}	0.00060	0.00058	
Material factor for Al conductors c_{th}	0.27	0.27	$\frac{10^{-18}\,m^4}{A^2 \cdot s}$
Factor for the thermal conductor expansion ε_{th}	0.000087	0.000090	
Factor for the elast. and therm. cond. expansion C_D	1.095	1.082	
Factor for dynam. deformation of the cond. curve C_F	1.05	1.05	
Horizontal span displacement b_h	1.01	1.06	m

The maximum value of the horizontal span displacement is found at the upper temperature and is 1.06 m. A centre-line distance of main conductors of a = 5 m means that the main conductors can approach to a minimum distance of 2.88 m in the most unfavourable case. As in Table 4-10, the required minimum conductor-conductor distance for the static case in a 420-kV system is 3.1 m. The permissible minimum distance in the event of a short circuit is therefore 1.55 m. Therefore, the strained conductors are short-circuit proof with reference to the horizontal span displacement, because 1.55 m $\leq$ 2.88 m.

Or otherwise expressed: the permissible horizontal span displacement is calculated at $b_{h\ zul}$ = (5m - 1.55 m) / 2 = 1.725 m. Because 1.725 m $\geq$ 1.06 m the conductors will not come too close in the event of a short circuit. The strained conductors are short-circuit proof.

4.2.4 Mechanical stress on cables and cable fittings in the event of short circuit

The forces occurring with a short circuit set the standard for the mechanical rating of the cable fittings. Even with stranded cables, these forces are very high because of the close proximity of the conductors. However, the forces are absorbed because they mostly act radially. A cable properly dimensioned thermally for short circuits is also suitable for withstanding mechanical short-circuit stresses.

The rated peak short-circuit currents i_p as per DIN VDE 0278 – 629-1 and – 629-2 must be verified at the end seals.

When short circuits occur, particularly high mechanical stresses occur with parallel single-conductor cables (Fig. 4-14).

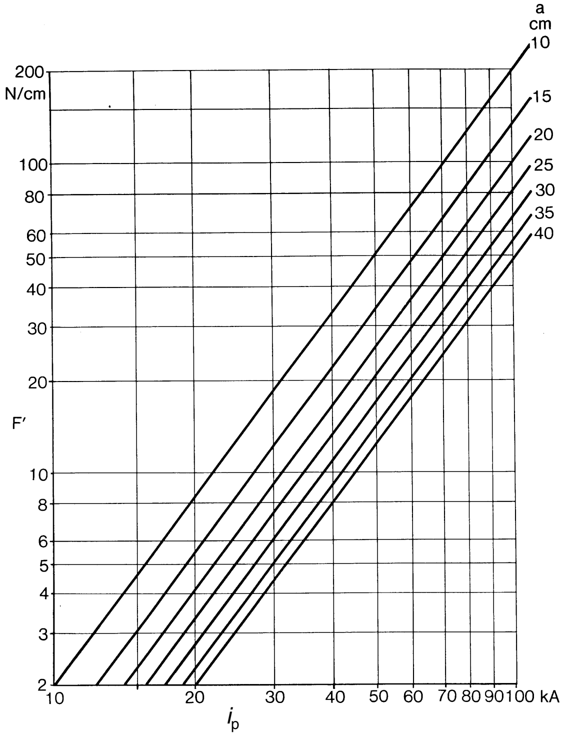


Fig. 4-14

Electrodynamic force density F' on two parallel single-conductor cables depending on the axis distance a of the cables and on the peak short-circuit current i_p.

With a three-phase short circuit, the effective forces are about 10 % lower than with a two-phase short circuit of the same current.

4.2.5 Rating the thermal short-circuit current capability

Busbars, including their feeders with the installed equipment (switches, current transformers, bushings), are also subject to thermal stress in the event of a short circuit. Verification is always required to ensure that they are sufficiently rated not only mechanically but also thermally for the short-circuit current.

The thermal stress depends on the quantity, the temporal sequence and the duration of the short-circuit current. A thermally equivalent short-time current I_{th} is defined as a current whose rms value generates the same amount of heat as another short-circuit current which may vary during the short-circuit duration T_k in its d.c. and a.c. components. It is calculated as follows for a single short-circuit event of the short-circuit duration T_k:

$$I_{th} = I''_k \cdot \sqrt{(m + n)}.$$

The factors m and n are determined as in Fig. 4-15. The effect of current limiting equipment can be taken into account. The individual values as in the above equation must be calculated for several sequential short-circuit durations (e.g. auto-reclosing). The resulting thermally equivalent phase fault current is then:

$$I_{th} = \sqrt{\frac{1}{T_k} \sum_{i=1}^{n} I_{thl}^2 \cdot T_{ki}} \text{ with } T_k = \sum_{i=1}^{n} T_{ki}.$$

The manufacturer provides the approved rated short-time withstand current I_{thr} and the rated duration of short circuit T_{kr} for equipment. This is the rms value of the current whose effect the equipment withstands during time T_{kr}.

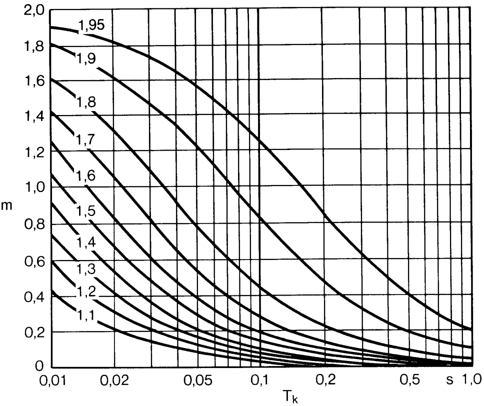
Electrical equipment has sufficient thermal resistance if:

$$I_{th} \leqq I_{thr} \text{ for } T_k \leqq T_{kr}$$

$$I_{th} \leqq I_{thr} \cdot \sqrt{\frac{T_{kr}}{T_k}} \text{ for } T_k \geqq T_{kr}.$$

T_k is the sum of the relay operating times and the switch total break time. Set grading times must be taken into account.

a)



b)

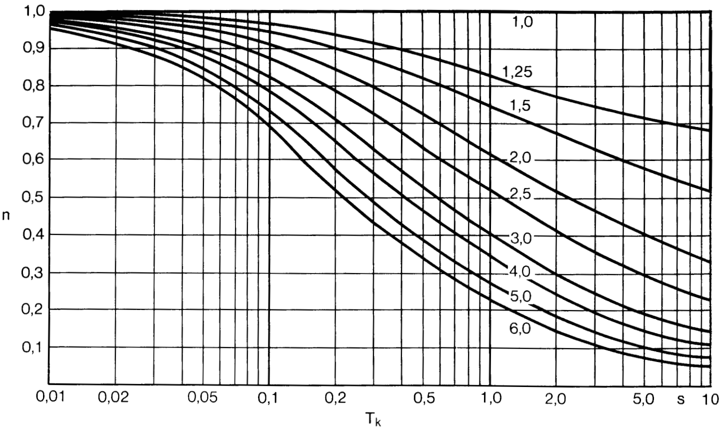
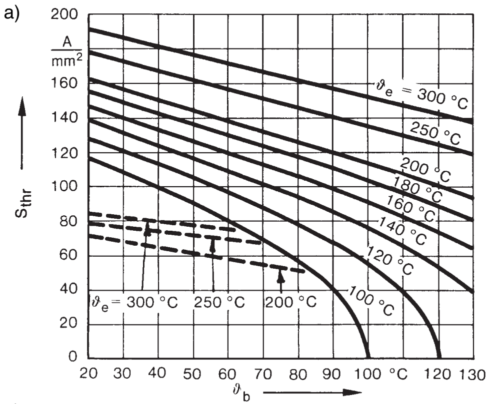


Fig. 4-15

Factors m and n for short-time current: a) factor m for the thermal effect of the direct current element with three-phase and single-phase alternating current at 50 Hz. Parameter: factor κ for calculating the peak short-circuit current i_p as in Fig. 3-2. At other frequencies f, the abscissa values for T_k must be multiplied by (50 Hz / f). b) factor n for the thermal effect of the alternating current element with three-phase and approximately with single-phase alternating current, parameter I''_k/I_k (see Fig. 3-1).

The equations of the curves for m and n are given in DIN EN 60865-1.

With line conductors, the thermally equivalent short-time current density S_{th} is used. It should be less than the rated short-time current density S_{thr}, which can be determined with Fig. 4-16.



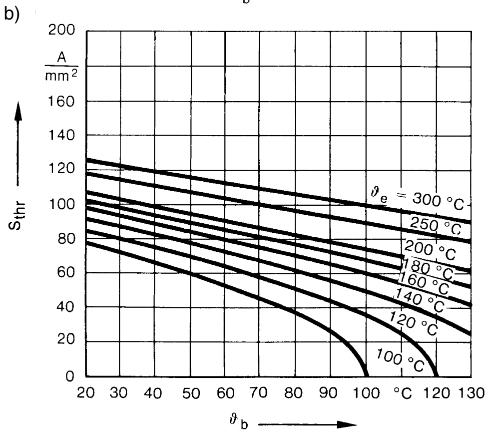


Fig. 4-16

Rated short-time current density S_{thr} for T_{kr} = 1 s: a) for copper (continuous curves) and unalloyed steel and steel cable (broken curves); b) for aluminium, Aldrey and Al/St.

The maximum continuous permissible operating temperature must be set as the temperature ϑ_b of a conductor, unless otherwise known (see Table 13-31 and 13-32). The end temperature ϑ_e of a conductor is the permissible conductor temperature in the event of a short circuit (see Tables 13-2, 13-3 and 13-32).

Bare conductors have sufficient thermal resistance when the thermally equivalent short-circuit current density conforms to the following equation:

$$S_{th} \leqq S_{thr} \cdot \sqrt{\frac{T_{kr}}{T_k}} \text{ for all } T_k.$$

Calculation example

The feeder to the auxiliary transformer of a generator bus must be checked for whether the cross section at 100 mm × 10 mm Cu and the current transformer are sufficient for the thermal stress occurring with a short circuit when the total break time $T_k = 1$ s. The installation must be rated for the following values:

$$I''_k = 174.2 \text{ kA}, \ \kappa = 1.8, \ I_k = 48{,}5 \text{ kA}, \ f = 50 \text{ Hz}.$$

For $\kappa = 1.8$ results $m = 0.04$ and for $\frac{I''_k}{I_k} = 3.6$ $n = 0.37$.

This yields

$$I_{th} = 174.2 \text{ kA} \sqrt{0.04 + 0.37} = 112 \text{ kA}.$$

According to the manufacturers, the rated short-time withstand current of the instrument transformer $I_{thr} = 125$ kA for $T_{kr} = 1$ s. The instrument transformers therefore have sufficient thermal strength.

The cross section of the feeder conductor is $A = 1000 \text{ mm}^2$.

Therefore, the current density is

$$S_{th} = \frac{112\,000 \text{ A}}{1000 \text{ mm}^2} = 112 \text{ A/mm}^2.$$

The permissible rated short-time current density at the beginning of a short circuit at a temperature $\vartheta_b = 80$ °C and an end temperature $\vartheta_e = 200$ °C as in Fig. 4-16:

$$S_{thr} = 125 \text{ A/mm}^2.$$

The feeder conductor therefore also has sufficient thermal strength.

The rated short-time current densities S_{thr} are given in Table 4-8 for the most commonly used plastic insulated cables.

The permissible rated transient current (1 s) for the specific cable type and cross section is calculated by multiplication with the conductor nominal cross section. The conversion is done with the following formula up to a short-circuit duration (T*k*) of max. 5 seconds:

$$I_{th}(T_k) = I_{thr} / \sqrt{T_k} \qquad T_k \text{ in seconds.}$$

Example

Permissible short-time current (break time 0.5 s) of cable N2XSY 1 × 240 RM/25, 12/20 kV:

$$I_{thr} = 240 \text{ mm}^2 \cdot 143 \text{ A/mm}^2 = 34.3 \text{ kA}$$

$$I_{th}(0.5 \text{ s}) = \frac{34.3 \text{ KA}}{\sqrt{0.5}} = 48.5 \text{ kA}$$

Note:

Short-time current densities for lower conductor temperatures at the beginning of the short circuit (cable only partially loaded) and values for mass-impregnated cables can be taken from DIN VDE 0276-620 and 0276-621 (HD 620 S1 and HD 621 S1).

Table 4-8

Permissible short-circuit conductor temperatures and rated short-time current densities for plastic-insulated cables

Insulation material	Nominal voltage U_0/U kV	Conductor temperature at beginning of the short circuit	Permissible end temperature	Conductor material	Rated short-time current density (1 s) A /mm²
PVC	0.6/1…6/10	70 °C	160 °C[1)]	Cu	115
				Al	76
			140 °C[2)]	Cu	103
				Al	68
XLPE	all ranges LV and HV	90 °C	250 °C[3)]	Cu	143
				Al	94

[1)] for cross sections ≤ 300 mm²
[2)] for cross sections > 300 mm²
[3)] not permitted for soldered connections

For extremely short break times with short circuits (T_k < 15 ms), current limiting comes into play and the thermal short-circuit current capability of carriers can only be assessed by comparison of the Joule integrals $\int i^2 \, dt = f(\hat{I}''_k)$. The cut-off power of the overcurrent protection device must be less than the still permissible heat energy of the conductor.

Permissible Joule integrals for plastic-insulated conducters:

A	= 1.5	2.5	4	10	25	50	mm²
$\int i^2 \, dt$	= $2.9 \cdot 10^4$	$7.8 \cdot 10^4$	$2.2 \cdot 10^5$	$1.3 \cdot 10^6$	$7.6 \cdot 10^6$	$3.3 \cdot 10^7$	A^2s

Current limiting overcurrent protection devices such as fuses or current limiting breakers are particularly advantageous for short-circuit protection of carriers. Their cut-off power in the event of a short circuit is small. As a result the Joule heat impulse $\int i^2 \, dt$ increases with increasing prospective short-circuit current I''_k with the zero-current interrupter many times faster than with the current limiter.

4.3 Dimensioning of wire and tubular conductors for static loads and electrical surface-field strength

4.3.1 Calculation of the sag of wire conductors in outdoor installations

Busbars and tee-offs must be rated for normal service current and for short circuit in accordance with DIN EN 60865-1, see Sec. 4.2.

Al/St wire conductors are primarily used for the tensioned busbars, for connecting equipment and tee-off conductors Al wire conductors with a similar cross section are used.

For wire data, see Sections 13.1.4, Tables 13-22 to 13-33.

Wire conductor sag is determined by the dead-end strings, the weight of the wire, the anticipated ice load, the supplementary load of tee-offs or fixed contacts for single-column disconnectors, by the wire-pulling force, by built-in springs or the spring stiffness of the supports and the cable temperature.

The wire conductor sag is calculated on the basis of the greatest sag occurring in the installation at a conductor temperature of + 80 °C, with very short span lengths possibly also at

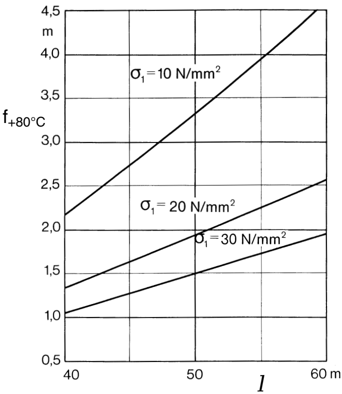


Fig. 4-17

Sag f for two-conductor bundles Al/St 240/40 mm², with 123-kV double endstrings, for spans of l = 40…60 m at conductor temperature +80°C. The following are included: two dead-end strings each 2.0 m in length, weight 80 kg (with 900 N ice load) and a tee-off of 10 kg in weight every 10 m. (Parameters of the family of curves: initial wire tension σ_1 at – 5 °C and normal ice load), f sag in m, l span length in m.

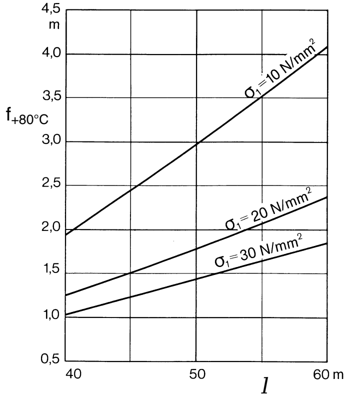


Fig. 4-18

Sag f for two-conductor bundles Al/St 300/50 mm², with 123-kV double endstrings, for spans of l = 40…60 m at conductor temperature +80°C. The following are included: two dead-end strings, each 2.0 m in length, weight 80 kg (with 900 N ice load) and a tee-off of 10 kg in weight every 10 m. (Parameters of the family of curves: initial wire tension σ_1 at – 5 °C and normal ice load), f sag in m, l span length in m.

As per DIN VDE 0210 the following applies:

– A distinction between the conductor with normal and increased supplementary load must be made. The ice load is designated with supplementary load. The normal supplementary load is assumed to be (5 + 0.1 d) N per 1 m of conductor or sub-conductor length. Here, d is the conductor diameter in mm[1]. The increased supplementary load is agreed depending on local conditions.

– For insulators, the normal supplementary load of 50 N per 1 m insulator string must be taken into account.

Typical values for a rough determination of the sags of tensioned busbars, tensioned and suspended wire links and lightning protection wires are given in Fig. 4-17 to 4-25.

[1] The normal supplementary load for conductors of 20 to 40 mm diameter corresponds to a layer of ice of 10 to 8 mm with a specific gravity of ice of 765 kg/m³.
In contrast, from January 2000 as per DIN VDE 0101 (HD 637 S1), ice thicknesses of 1, 10 or 20 mm with a specific gravity of ice of 900 kg/m³ will be assumed.

Sag of the tensioned busbars with loads, dead-end strings and tee-offs at every 10 m (width of bay) with a weight of 10 kg each

The sags and tensions of the busbar wires are influenced by their dead-end strings and tee-offs (point loads).

The busbar sags in a 123-kV outdoor installation with a bay width of 10.0 m can be roughly determined using the diagrams in Figs. 4-17 to 4-20. These give the most common types of wire conductors like two-conductor bundle 240/40 mm², two-conductor bundle 300/50 mm², single-conductor wire 380/50 mm² and single-conductor wire 435/55 mm², for spans of 40…60 m and initial wire tensions σ_1 = 10.0…30.0 N/mm² with ice load as per DIN VDE 0210, values for the sags occuring at + 80 °C conductor temperature. This ice load is (5 + 0.1 d) N/m with wire diameter d in mm.

At 245- and 420-kV outdoor installations in diagonal arrangement with single-column disconnectors the busbars take the weight of the disconnector fixed contacts instead of the tee-off wires. To limit the temperature-dependent change in sag, spring elements are frequently included in the span to maintain the suspended contacts within the reach of the disconnector scissors.

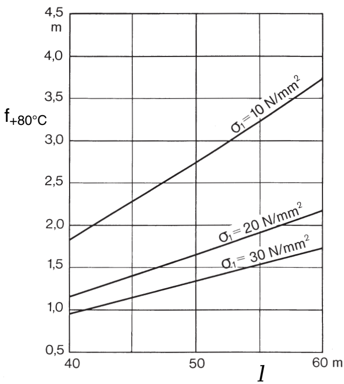


Fig. 4-19

Sag f for single-conductor wires Al/St 380/50 mm², with 123-kV double-end strings, for spans of l = 40…60 m at conductor temperature + 80°C. The following are included: two dead-end strings, each 2.0 m in length, weight 80 kg (with 900 N ice load) and a tee-off every 10 m of 10 kg in weight. (Parameters of the family of curves initial wire tension σ_1 at – 5 °C and normal ice load), f sag in m, l span length in m.

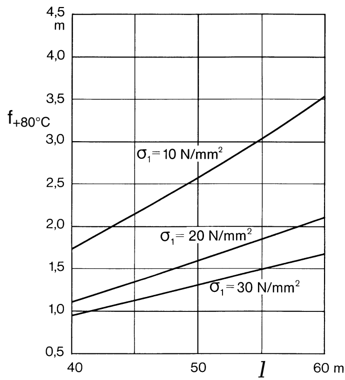


Fig. 4-20

Sag f for single-conductor wires Al/St 435/55 mm², with 123-kV double-end strings, for spans of l = 40…60 m at conductor temperature + 80°C. The following are included: two dead-end strings, each 2.0 m in length, weight 80 kg (with 900 N ice load) and a tee-off every 10 m of 10 kg in weight. (Parameters of the family of curves initial wire tension σ_1 at – 5 °C and normal ice load), f sag in m, l span length in m.

Sag of the spanned wire conductors

In many outdoor installations spanned wire conductors with dead-end strings are required. They generally only have a wire tee-off at the ends of the stays (near the string insulators).

The sag can be calculated as follows when σ_x is known:

$$f_x = \frac{g_n}{2 \cdot \sigma_x \cdot A} [m' \cdot (0.25\, l^2 - l_k^2) + m_k \cdot l_k]$$

f_x sag m, σ_x horizontal component of the cable tension N/mm², m' mass per unit length of wire kg/m, with ice load if applicable, m_K weight of insulator string in kg, A conductor cross section in mm², l span including insulator strings in m, l_k length of the insulator string in m, g_n gravity constant. The sags of some wire conductor spanned with double-end strings in 123 and 245-kV switchgear installations can be taken from the curves in Fig. 4-21 as a function of the span.

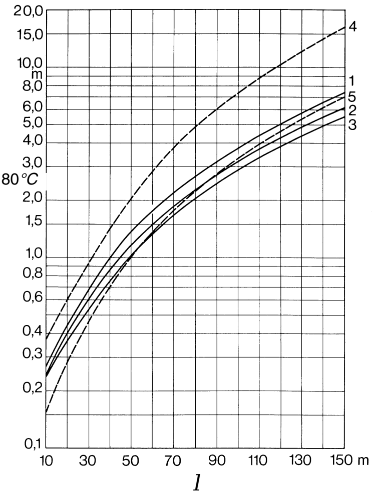


Fig. 4-21

Sag $f_{80\,°C}$ for spanned wire connections for spans up to 150 m with conductor temperature + 80 °C:

1 two-conductor bundle Al/St 560/50 mm², 245-kV-double-end strings, σ_1 20,0 N/mm² at – 5 °C and normal ice load

2 two-conductor bundles Al/St 380/50 mm², 245-kV-double-end strings, σ_1 30.0 N/mm² at – 5 °C and normal ice load

3 two-conductor bundles Al/St 240/40 mm², 245-kV-double-end strings, σ_1 40.0 N/mm² at – 5 °C and normal ice load

4 two-conductor bundles Al/St 240/40 mm², 123-kV-double-end strings, σ_1 10.0 N/mm² at – 5 °C and normal ice load

5 two-conductor bundles Al/St 435/50 mm², 123-kV-double-end strings, σ_1 20.0 N/mm² at – 5 °C and normal ice load

(sag in logarithmic scale)

Fracture of an insulator of a double dead-end string

For safety reasons the wire connections in switchgear installations have double dead-end strings. The fracture of an insulator results in an increase in the sag in the middle of the span.

The greatest sag f_k is roughly calculated as follows

$$f_k = \sqrt{f_\vartheta^2 + \frac{3}{8} \cdot 0{,}5\, y \cdot l}$$

f_ϑ = sag at ϑ °C
l = span length
y = length of yoke of double-end string

The curves in Fig. 4-22 can be used to make an approximate determination for $y = 0.4$ m of the greatest occurring sags.

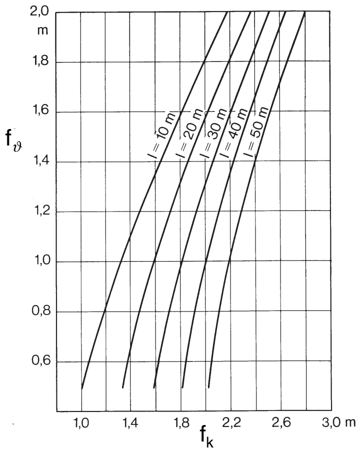


Fig. 4-22

General determination of changes in sag in the event of a fracture of an insulator of the double-end spring. Length of yoke between two insulators $y = 0.4$ m, f_k maximum sag in m, f_ϑ sag at ϑ °C in m, parameter l length of span.

Sag of the earth wire

Outdoor installations are protected against lightning strikes by earth wires. Al/St wires are generally used. Section 5.4 shows the configuration and the protection range of the earth wires in detail. They are placed along the busbar and at right-angles to the overhead line and transformer feeder bays.

The ice load on the wires must also be considered here. For Al/St 44/32 and Al/St 50/30 earth wires in Fig. 4-25, the sags can be determined at conductor temperature + 40 °C (because there is no current heat loss) and for span lengths to 60 m at cable tensions $\sigma_1 = 10.0$ to 30.0 N/mm^2. In practice, the earth wires are generally spanned so their sag is identical to that of the busbars.

Wire connections of equipment

In outdoor installations the high-voltage equipment is generally connected with wire condcutors. The applicable wire pull depends on the approved pull (static + dynamic) of the apparatus terminals. The minimum clearances and conductor heights over walkways in switchgear installations are specified in Section 4.6. These are minimum dimensions. For rating for mechanical short-circuit current capability, see Section 4.2.

The sags and conductor tensions can be calculated with standard formulae used in designing overhead lines. The sag in midspan is calculated with the parabolic equation:

$$f_x = \frac{(m' g_n + F_z)\, l^2}{8 \cdot \sigma_x \cdot A}$$

f_x sag in m
A cond. cross section mm²
l span in m
σ_x horizontal component of the cond. tension N/mm²
m' conductor weight per unit length in kg/m
F_z normal ice load in N/m (in DIN VDE 0210 designated as supplementary load). $F_z = (5 + 0.1\ d)$ N/m.

Values for DIN wire conductors, see Section 13.1.4, Tables 13.22 to 13.29.

Tensions in wire connections

For the conductor sag of 0.5 m accepted in practice at + 80 °C conductor temperature, the required tensions depending on the span for the Al wire conductor cross sections 240, 300, 400, 500, 625 and 800 mm² can be taken from the curves in Figs. 4-23 and 4-24. The permissible mechanical terminal load of the installed devices and apparatus must be observed.

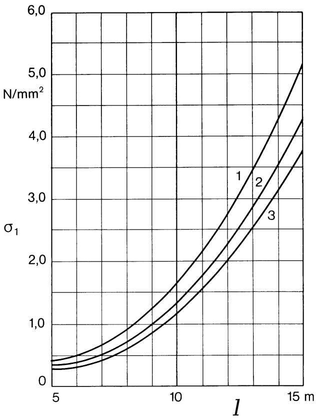


Fig. 4-23

Tensions σ_1 for suspended wire connections at –5 °C and normal ice load: 1 cable Al 240 mm²; 2 cable Al 400 mm², 3 cable Al 625 mm²

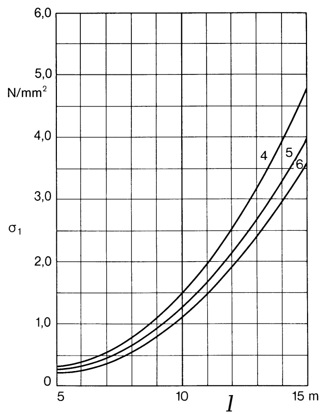


Fig. 4-24

Tensions σ_1 for suspended wire connections at –5 °C and normal ice load: 4 cable Al 300 mm²; 5 cable Al 500 mm², 6 cable Al 800 mm²

Sag in proximity to terminal points

When connecting the rotary disconnector, ensure that the cable sag does not affect the functioning of the disconnector arm. As shown in Fig. 4-26, the sag determines the minimum height of the conductor at the distance *c* from the terminal point *A*. The sag at distance *c* is calculated as follows:

$$f_c = \frac{4 \cdot f_{max} \cdot c \cdot (l - c)}{l^2}$$

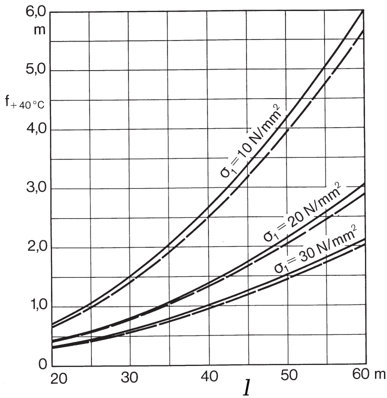


Fig. 4-25

Sag f for earth wire Al/St 44/32 mm² ——— and Al/St 50/30 mm² — — — for spans of 20 to 60 m at conductor temperature + 40 °C (no Joule heat). (Parameters of the family of curves: initial tension σ_1 at –5 °C and normal ice load), f sag in m, l span length in m.

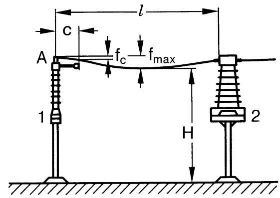


Fig. 4-26

Sag of a connection of equipment at distance c from terminal point A. 1 rotary disconnector, 2 current transformer, A terminal point, l length of device connection, f_{max} sag in midspan, f_c sag at distance c, H height above ground (see Fig. 4-37).

4.3.2 Calculation of deflection and stress of tubular busbars

In general, the deflection *f* and the stress σ of a tube is the result of its own weight

$$f = \frac{1}{i} \cdot \frac{Q \cdot l^3}{E \cdot J} \text{ and } \sigma = \frac{k \cdot Q \cdot l}{W}$$

Where:

$Q = m' \cdot g_n \cdot l$	load by weight of the tube between the support points
l	span (between the support points)
E	module of elasticity (for copper = 11 · 10⁶, for Al = 6.5…7.0 · 10⁶, for steel = 21 · 10⁶, for E-AlMgSi 0.5 F 22 = 7 · 10⁶ N/cm²; see Table 13-1)

J	moment of inertia (for tube $J = 0.049\,[D^4 - d^4]$) as in Table 1-22
W	moment of resistance for bending (for tube $W = 0.098\,[D^4 - d^4]/D$) as in Table 1-22
m'	weight of tube per unit of length (without supplementary load) in kg/m (see Tables 13-5, 13-9 and 13-10)
g_n	gravity constant 9.81 m/s²
i, k	factors (see Table 4-9)

Table 4-9

Factors for calculating the deflection of tubular busbars

Type of support	*i*	*k*
Tube supported at both ends	*77*	*0.125*
Tube one end fixed, one freely supported	*185*	*0.125*
Tube fixed at both ends	*384*	*0.0834*
Tube on three support points	*185*	*0.125*
Tube on four support points	*145*	*0.1*
Tube on more than four support points	*130*	*0.11*

As per DIN VDE 0101, an ice load equivalent to a layer of ice of 1.5 cm with a specific gravity of 7 kN/m³ must be taken into account (see footnote [1]) on page 151). When doing the calculation with ice, the load Q (due to the weight of the tube) must be increased by adding the ice load.

A permissible value for the compliance is only available as a typical value for optical reasons. For the compliance under own weight, this is l /150 or D and for the compliance under own weight and ice l / 80.

Permissible value for the stress under own weight plus ice is $R_{p0.2}$ / 1.7 with $R_{p0.2}$ as in Table 13-1. Permissible value with simultaneous wind load is $R_{p0.2}$ / 1.5.

Example:

Given an aluminium tube E-AlMgSi 0.5 F 22 as in Table 13-10, with external diameter 80 mm, wall thickness 5 mm, span 8 m, supported at both ends. Then

$$Q = m' \cdot g_n \cdot l = 3.18\,\frac{\text{kg}}{\text{m}} \cdot 9.81\,\frac{\text{m}}{s^2} \cdot 8\text{ m} = 250\text{ N}$$

$$J = 0.049\,(8^4 - 7^4)\text{ cm}^4 = 83\text{ cm}^4$$

$$W = 0.098\,\frac{(8^4 - 7^4)}{8}\text{cm}^3 = 20.8\text{ cm}^3$$

The deflection is:

$$f = \frac{1}{77} \cdot \frac{250\text{ N} \cdot 8^3 \cdot 10^6\text{ cm}^3}{7 \cdot 10^6\,(\text{N/cm}^2) \cdot 83\text{ cm}^4} = 2{,}9\text{ cm}$$

The stress is:

$$\sigma = \frac{0.125 \cdot 250\text{ N} \cdot 800\,\text{cm}}{20.8\text{ cm}^3} = 12\,\frac{\text{N}}{\text{mm}^2}$$

Deflection and stress are acceptable.

4.3.3 Calculation of electrical surface field strength

The corona effect on the conductor surface of overhead lines is a partial electrical discharge in the air when the electrical field strength exceeds a critical value on the conductor surface.

There is no specification for the permissible surface field strength for outdoor installations. In general for overhead lines, the value is 16…19 kV/cm, in individual cases up to 21 kV/cm is approved. These values should also be retained with switchgear installations. The surface field strength *E* can be calculated with the following formula:

$$E = \frac{U}{\sqrt{3}} \cdot \frac{\beta}{r_L \cdot \ln\left(\frac{a}{r_e} \cdot \frac{2 \cdot h}{\sqrt{4\,h^2 + a^2}}\right)}$$

where $$\beta = \frac{1 + (n-1)\; r_L / r_T}{n}$$

$$r_e = \sqrt[n]{n \cdot r_L \cdot r_T^{\,n-1}}$$

$$r_T = \frac{a_T}{2 \cdot \sin(\pi / n)}$$

The following apply in the equations:

- E electrical surface field strength
- U nominal voltage
- β multiple conductor factor (for tube = 1)
- r_L conductor radius
- r_T radius of the bundle
- r_e equivalent radius of bundle conductor
- a_T centre-to-centre distance of sub-conductors
- a centre-to-centre distance of main conductors
- h conductor height above ground
- n number of sub-conductors per bundle

Example:

Lower busbars in a 420-kV outdoor installation with Al/St 4 × 560/50 mm², as in Fig. 3-17a, Section 3.4.4, at a medium height of 9.5 m above ground: U = 380 kV, r_L = 1.61 cm, a_T = 10 cm, a = 500 cm, h = 950 cm, n = 4. With these figures, the above equations yield:

$$r_T = \frac{10\text{ cm}}{2 \cdot \sin\frac{\pi}{4}} = 7.07\text{ cm}$$

$$r_e = \sqrt[4]{4 \cdot 1.61 \cdot 7.07^3} = 6.91\text{ cm}$$

$$\beta = \frac{1 + (4-1)\;\frac{1.61}{7.07}}{4} = 0.42$$

$$E = \frac{380\text{ kV}}{\sqrt{3}} \cdot \frac{0{,}42}{1.61\text{ cm}\,\ln\left(\frac{500}{6.91} \cdot \frac{2 \cdot 950}{\sqrt{4 \cdot 950^2 + 500^2}}\right)} = 13.5\,\frac{\text{kV}}{\text{cm}}$$

The calculated value is within the permissible limits. This configuration can be designed with these figures.

4.4 Dimensioning for continuous current rating

4.4.1 Temperature rise in enclosed switch boards

Electrical equipment in switchboards gives off loss heat to the ambient air. To ensure fault-free function of this equipment, the specified limit temperatures must be retained inside the switchboard.

The following applies according to the relevant IEC or VDE specifications

- with open installations as ambient temperature the temperature of the ambient room air (room temperature ϑ).
- in closed installations as ambient temperature the temperature inside the enclosure (inside air temperature ϑ_i).
- as temperature rise the difference between inside air temperature (ϑ_i) and room air temperature (ϑ).

The most significant heat sources inside the enclosure are the conducting paths in the main circuit. This includes the circuit-breakers and fuses, including their connections and terminals and all the auxiliary equipment in the switchboard.

Inductive heat sources such as eddy currents in steel parts only result in local temperature rises. Their contribution is generally negligible for currents < 2500 A.

The power dissipation for the electrical equipment can be found in the relevant data sheets.

In fully enclosed switchboards (protection classes above IP 50) the heat is dissipated to the outside air primarily by radiation and external convection. Thermal conduction is negligibly small.

Experiments have shown that in the inside temperature is distributed depending on the height of the panel and on the equipment configuration. The density variations of the heated air raises the temperature in the upper section of the enclosure.

The temperature distribution can be optimized when the electrical equipment with the greatest power dissipation is positioned in the lower part of the panel, so the entire enclosure is involved in heat dissipation as far as possible.

When installed on a wall, the panel should have 8…10 cm clearance from the wall. This allows the rear wall of the panel to be involved effectively in dissipating heat.

The average air temperature inside the enclosure, neglecting the heat radiation, can be calculated as follows:

$$\Delta \vartheta = \frac{P_{\mathrm{V\,eff}}}{\alpha \cdot A_{\mathrm{M}}}$$

$\Delta \vartheta$ Temperature increase of air inside enclosure

$P_{\mathrm{V\,eff}}$ power dissipation with consideration of load factor as per DIN EN 60439-1 (VDE 0660 Part 500) Tab. 1

A_{M} heat-dissipating surface of enclosure

α Heat transfer coefficient:

6 W/(m$^2 \cdot K$) if sources of heat flow are primarily in the lower half of the panel,

4.5 W/(m$^2 \cdot K$) where sources of heat flow are equally distributed throughout the height of the panel,

3 W/(m$^2 \cdot K$) if sources of heat flow are primarily in the upper half of the panel.

If there are air vents in the enclosure, such as with IP 30, heat dissipation is primarily by convection.

The heat transfer from the air in the interior of the enclosure to the ambient air is much better in this case than with fully enclosed designs. It is influenced by the following:

- the size of the panel,
- the ratio of air outlet and inlet vents to the entire heat-dissipating surface,
- the position of air inlets and outlets,
- the distribution of heat sources inside the panel and
- the temperature difference.

The internal air temperature will be in the range of 0.5 to 0.7 times of that calculated in the above equation.

If switchgear assemblies develop higher heat loss or if they have a non-linear flow model, they must be equipped with internal fans to force the heat generated out to the surrounding space. An external room ventilation system will then be required to extract the heat from the switchgear room.

VDE specifies + 40 °C as the upper limit for the room temperature and – 5 °C for the lower limit.

The electrical equipment cannot be applied universally above this range without additional measures. Excessive ambient temperatures at the devices affects functioning or load capacity. The continuous current cannot always be fully used, because a room temperature of + 40 °C does not leave sufficient reserve for the overtemperature inside the enclosure.

The assessment must be based on the assumption that the overtemperatures set in VDE 0660 Part 500 Tab. 3 should not be exceeded and that the equipment will operate properly.

Example:

Panel in protection class IP 54, fitted with 12 inserts. Every insert has fuses, air-break contactors and thermal overcurrent relays for motor control units. Heat flow sources are evenly distributed throughout the height of the panel.

power dissipation P_v = 45 W per insert.
load factor a = 0.6 (as per VDE 0660 Part 500 Tab. 1)
heat-dissipating enclosure surface $A_M = 4\ \text{m}^2$.

With the stated component density, a check is required to ensure that the electrical equipment is subject to a maximum operating temperature of 55 °C. Room temperature $\vartheta = 35$ °C.

Effective power dissipation $P_{V\,eff} = a^2 \cdot P_V = 0.6^2 \cdot 12 \cdot 45\ \text{W} = 194.4\ \text{W}$.

$$\Delta\vartheta = \frac{P_{V\,eff}}{\alpha \cdot A_M} = \frac{194.4\ \text{W} \cdot \text{m}^2\ \text{K}}{4.5\ \text{W} \cdot 4\ \text{m}^2} = 10.8\ \text{K}$$

$$\vartheta_i = \vartheta + \Delta\vartheta = 35 + 10{,}8 = 45.8\ ^\circ\text{C}.$$

For additional details on determining and assessing the temperature rise in switchboards, see DIN EN 60439-1 (VDE 0660 Part 500) Section 8.2.1 and Section 7.3 of this publication.

4.4.2 Ventilation of switchgear and transformer rooms

Design criteria for room ventilation

The air in the room must meet various requirements. The most important is not to exceed the permissible maximum temperature. Limit values for humidity and air quality, e.g. dust content, may also be set.

Switchboards and gas-insulated switchgear have a short-term maximum temperature of 40 °C and a maximum value of 35°C for the 24h average. The installation requirements of the manufacturers must be observed for auxiliary transformers, power transformers and secondary installations.

The spatial options for ventilation must also be considered. Ventilation cross sections may be restricted by auxiliary compartments and buildings. If necessary, the loss heat can be vented through a chimney. If HVAC (air-conditioning) installations and air ducts are installed, the required space and the configuration must be included at an early stage of planning.

Ultimately, economic aspects such as procurement and operating expenses must be taken into account as well as the reliability (emergency power supply and redundancy) of the ventilation.

At outside air temperatures of up to 30 °C, natural ventilation is generally sufficient. At higher temperatures there is danger that the permissible temperature for the equipment may be exceeded.

Figs. 4-27 and 4-28 show frequently used examples of room ventilation.

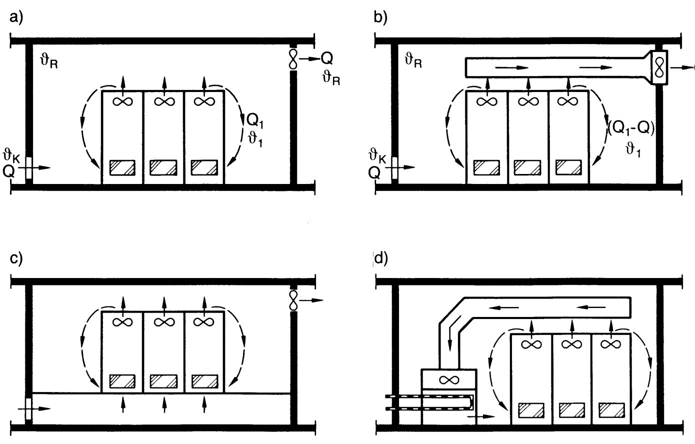


Fig. 4-27

Compartment ventilation: a) Simple compartment ventilation, b) compartment ventilation with exhaust hood above the switchboard, c) ventilation with false floor, d) ventilation with recirculating cooling system

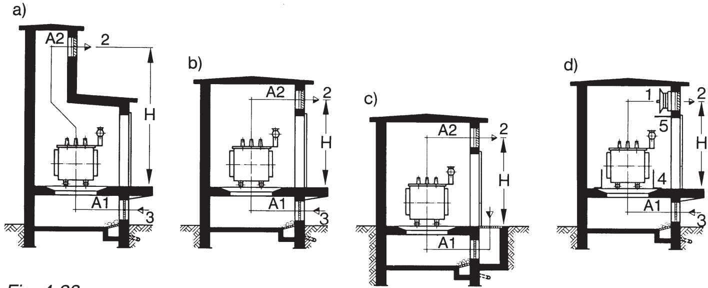


Fig. 4-28

Cross section through transformer cells:

a) incoming air is channelled over ground, exhaust air is extracted through a chimney. b) as in a), but without chimney. c) incoming air is channelled below ground, exhaust air is removed through an opening in the wall of the transformer compartment. d) transformer compartment with fan. A_1 = incoming air cross section, A_2 = exhaust air cross section, H = "chimney" height, 1 = fan, 2 = exhaust air slats, 3 = inlet air grating or slats, 4 = skirting, 5 = ceiling.

The ventilation efficiency is influenced by the configuration and size of the incoming air and exhaust air vents, the rise height of the air (centre of incoming air opening to centre of exhaust air opening), the resistance in the path of the air and the temperature difference between incoming air and outgoing air. The incoming air vent and the exhaust air vent should be positioned diagonally opposite to each other to prevent ventilation short circuits.

If the calculated ventilation cross section or the chimney opening cannot be dimensioned to ensure sufficient air exchange, a fan will have to be installed. It must be designed for the required quantity of air and the pressure head.

If the permissible room temperature is only slightly above or even below the maximum outside temperature, refrigeration equipment or air-conditioning is used to control the temperature.

In ventilated and air-conditioned compartments occupied by personnel for extended periods the quality regulations for room air specified by DIN 1946 must be observed.

The resistance of the air path is generally:

$$R = R_1 + m^2 R_2.$$

Here: R_1 resistance and acceleration figures in the incoming air duct, R_2 resistance and acceleration figures in the exhaust air duct, m ratio of the cross section A_1 of the incoming air duct to the cross section A_2 of the exhaust air duct. Fig. 4-28 shows common configurations.

The total resistance consists of the components together. The following values for the individual resistance and acceleration figures can be used for an initial approximation:

acceleration	1	slow change of direction	0…0.6
right-angle bend	1.5	wire screen	0.5…1
rounded bend	1	slats	2.5…3.5
a bend of 135 °	0.6	cross section widening	0.25…0.9[1)]

[1)] The smaller value applies for a ratio of fresh air cross section to compartment cross section of 1:2, the greater value for 1:10.